LONG RUN DEVIATIONS FROM THE PURCHASING POWER PARITY BETWEEN GERMAN MARK AND U.S. DOLLAR. OIL PRICE - THE MISSING LINK?

Markus Lahtinen

Working Paper 19
September 2003
The aim of this paper is to identify and investigate empirically the long-run determinants of real exchange rate fluctuations between Germany and the United States since the collapse of Bretton Woods system. This study uses the multivariate cointegration technique to find steady state relations of real exchange rates. Possible I(2) property of the data generating process is also examined using the latest statistical tools.

Number of studies have furnished fairly persuasive evidence that the deviations from purchasing power parity (PPP) derive in large part from differences in relative traded goods prices especially if the U.S. dollar is included into the estimation vector. These findings have led some researcher to suggest that there might be an unidentified real or financial factor causing persistent shifts in the real exchange rate. In this study we have used a more explicit explanation for this exchange rate shock. Real oil price is shown to be an important factor modeling the real exchange rate movements between German mark and U.S. dollar. In the presence of sunk cost of arbitrage uncertainty as to the permanence of the shocks causing relative price changes will widen the range within which price differentials can fluctuate and also increase the time before arbitrage commences. Oil price is shown to be the important source of uncertainty, which will appreciate US dollar without the price arbitrage of tradable goods. When the oil price shock is included into the estimation vector, we can also identify the classical Harrod-Balassa-Samuelson condition not generally found in literature between the German mark and the US dollar.

**KEY WORDS:** Purchasing power parity, oil price, cointegration
1. INTRODUCTION

The purchasing power parity (hereafter PPP) hypothesis states that national price levels expressed in a common currency should be equal. The empirical evidence is, however, rather contradictory. Recent evidence suggests, however, that the failures of the law of one price are not only significant, but they also play a dominant role in the behavior of real exchange rate. The key to resolving the possible failure of PPP for a recent floating period lies in understanding the forces that keep exchange rate away from an equilibrium based solely on relative prices.

The aim of this paper is to identify and investigate empirically the long-run determinants of real exchange rate fluctuation between the United States and Germany since the collapse of the Bretton Woods system of fixed exchange rates in the early 1970’s. The real exchange rate is measured using CPI deflators. The bilateral exchange rate between the United States and Germany has been chosen in order to fully understand the complicated interrelationship between USA and Europe. This is extremely important because the advent of the Euro means that Europe will be the crucial continent in the international monetary system. Because of the different economic structure of the Euro-area relative to Germany and also the different operational objectives of the European Central Bank relative to the Bundesbank, the Euro may, of course, have different properties compared to the German mark. Nevertheless, there are good reasons to believe that the study of the dollar-mark system is useful both in terms of indicating how the relationship between two large open economies may be modeled and also in suggesting how such a system may behave in terms of its long and short run properties.

Although the number of previous attempts at modeling real exchange rates for recent floating period has been enormous, such a work has not always proved particularly fruitful. Bilateral exchange rate models fail typically to establish a significant long-run link between the real exchange rate and fundamentals. However, one key message to come from the modeling exchange rates is that the econometric method used can have a crucial bearing on the findings of significant and sensible long-run relationships. This paper uses the cointegration technique developed by Johansen (1988, 1991) to find steady state relations of real exchange rates. The Johansen maximum likelihood approach has been shown to be more efficient way to analyze cointegration structure relative to traditional Engle-Granger methodology.¹ Allowing modeling in a multivariate framework is one of the most crucial advantages of the Johansen maximum likelihood approach. Furthermore, the

¹ See the discussion in Maddala and Kim (1998).
possibility to impose restrictions on the cointegration vectors is especially important if the model contains more than one cointegration vector.

One important theme in this paper concerns the importance of distinguishing between statistical and theoretical measures of a unit root. There are many arguments in favor of considering a unit root (a stochastic trend) as a convenient econometric approximation rather than as a deep structural parameter. For example, in many empirical studies inflation is usually assumed to be stationary process. Although this is a theoretically only acceptable hypothesis in the long run time perspective, it is not necessary suitable statistical formulation in the medium run, i.e. a period such as the recent float after Bretton Woods. For a such period prices are found to be even I(2), implying that inflation rates are I(1). Possible I(2) property of the data generating process is examined using the latest statistical tools developed by Johansen (1995), Paruolo (1996) and Rahbek, Kongsted and Jørgensen (1998). This analysis gives us a possibility to examine more closely the relationship between the prices and nominal exchange rate.

Another important issue concerns the persistence in the real exchange rates. A number of studies have demonstrated that for the recent floating experience real exchange rates are I(1) process. This evidence suggests that there are also exogenous real shocks which may affect the deviations from the PPP. If the sources of PPP disturbances are real in nature, we would argue this will have a permanent effect on the real exchange rate opposite to possibility that the predominant force upsetting the PPP relationship is nominal which will have only a transitory effect on deviations from PPP if money is assumed to be neutral in the long run (as in the celebrated Dornbusch overshooting model).

Sectoral productivity differentials across countries have long been suggested as major determinants of real exchange rate movements in the long run. This effect, known as the Harrod-Balassa-Samuelson (hereafter HBS) effect, is a tendency for countries with higher productivity in tradables compared with non-tradables to have higher price levels. The HBS hypothesis divides real exchange rate movements into two components. The first component of the hypothesis is the assumption that the relative price of non-tradables is proportional to the ratio of labor products and

---

2 As a survey, see Rogoff (1996). Recently, the importance of non-linearity as a source of non-stationarity is emphasized by Obstfeld and Taylor (1997), and Michale, Nobay and Peel (1997).
3 See Balassa (1964) and Samuelson (1964).
the second component is the assumption of PPP for traded goods. These two components combine to produce a simple model of real exchange rate movements.

Recently, a number of studies, such as Asea and Mendoza (1994) and De Gregorio and Wolf (1994), have furnished fairly persuasive evidence that, at least for industrial countries, deviations from the PPP derive large part from differences in relative traded prices across countries. Engel’s (1999) results concerning the U.S. dollar are similar and they suggest that consumers prices for tradables goods behave very much in the same way as non-tradables consumer prices. These results are quite puzzling in a view of HBS theory which holds that the relative tradables prices show little long-term variation across countries compared with the variation in relative non-tradables prices. However, as discussed in MacDonald (2000), this does not necessarily imply that the HBS effect is in itself unimportant or insignificant, but the above evidence may suggest that the dominant component of real exchange rate behavior is nominal exchange rate changes even in a long run.

Since Krugman’s (1987) article, studies on prices and exchange rates have also focused intensively on the issues of markup adjustment. Moving outside the competitive markets paradigm, pricing the market behavior give rise to impediments to goods arbitrage. Another intuitively appealing concept for incomplete exchange rate pass-through, i.e. the less than proportional response of import prices to changes in exchange rate, is given by the international real option investment theory, inspired by Dixit (1989a and b). We discuss the real option investment theory as an explanation for incomplete exchange rate pass-through even in a competitive market paradigm.

Uncertainty is a necessary condition for large and persistent deviations from the PPP in a real option approach. We introduce oil price as an important source of uncertainty, which creates a lasting price differential between price indices through incomplete pass-through. It also appreciates the nominal US dollar exchange rate relative to the German mark. The importance of oil price for US exchange rate movements has been argued by Krugman (1983). This model builds on long-run changes on the balance of payment due to the change in oil price using a multi country framework (Germany, the United States and OPEC).

The relationship between oil price and real exchange rate has also been examined, for example, in Johansen and Juselius (1992), Rogoff (1992), MacDonald (1997), Chaudhuri and Daniel (1998), and Amano and Van Norden (1998). Johansen and Juselius introduce oil price as an exogenous but significant variable for the PPP relation of United Kingdom. Rogoff (1992) finds oil price a significant variable for the Japanese yen-the U.S. dollar real exchange rate. However, this finding depends on the chosen time period. MacDonald (1997) finds weak support for the importance of oil price using a multivariate cointegration method for the real effective exchange rates of the U.S. dollar, the German mark, and the Japanese Yen. Chaudhuri and Daniel (1998), and Amano and Van Norden (1998) use real effective exchange rate of the U.S. dollar and find oil price significant variable even in a bivariate system.
2. TRADABLE AND NON-TRADABLE GOODS

The real exchange rate is a measure of one country’s overall price level relative to another country’s. The real exchange rate, defined with respect to a general or overall price level, such as the CPI, is given by

\[ q_t = p_t - p_t^* - s_t \]  

(2.1)

where \( q_t \) denotes the logarithm of real exchange rate, \( p_t \) denotes the log of the domestic price level, \( p_t^* \) the log of the foreign price level and \( s_t \) the log of the nominal exchange rate defined as the home currency price of a unit of foreign currency. In this context, therefore, a rise in \( q_t \) denotes an appreciation of the real exchange rate. To measure the price level, we decompose it into the traded and non-traded components and use a geometric average of these prices in both country

\[ p_t = (1 - \alpha) p_t^r + \alpha p_t^N, \quad \alpha < 1 \]  

(2.2)

where \( p_t \) denotes the logarithm of the price index, \( p_t^r \) is the log of the traded goods price index, \( p_t^N \) is the log of the non-traded goods price index and \( \alpha \) is the share that nontraded goods take in the price index. Letting an asterisk represent the foreign country, one can also write

\[ p_t^* = (1 - \beta) p_t^r + \beta p_t^N^*, \quad \beta < 1 \]  

(2.3)

where \( \beta \) is nontraded good’s share in the foreign price index. Thus, following Engel (1999), the real exchange rate can be written as

\[ q_t = x_t + y_t, \]  

(2.4)

where

\[ x_t = p_t^r - p_t^r^* - s_t \]  

(2.5) and

\[ y_t = \alpha (p_t^N - p_t^r) - \beta (p_t^N^* - p_t^r^*). \]  

(2.6)
\( y_t \) is a traditional Harrod-Balassa-Samuelson condition, which relates labor productivity to nontradable goods prices and \( x_t \) defines the PPP condition for tradable goods.\(^5\) Non-constancy of the real exchange rate for traded goods may arise if the kinds of goods entering international trade are imperfect substitutes and there are factors which introduce systematic variability into \( y_t \).

2.1. NON-TRADABLE PRICES

In order to define HBS we have to assume perfect international competition of goods and capital markets to ensure that the prices of tradables and interest rate are pinned down.\(^6\) The former then determinates uniquely the wage rate of internationally immobile assumed labor by equalization of marginal product and the given world price. This, with given intersectional factor mobility (labor and capital), means that relative prices are set exclusively by the level of productivity in the two sectors, i.e. the productivity in the non-tradable (tradable) sector then determines the price of non-tradables (tradable).

Since labor and capital factors are free to move between sectors costlessly, only supply side factors matter.\(^7\) In HBS model demand side factors will affect the real exchange rate only if the assumption of perfect competition, PPP for traded goods, or perfect capital mobility is relaxed. The conclusion is, ceteris paribus, that economies with a higher level of productivity in tradables will thus be characterized by higher wages and also by higher price of non-tradables if productivity in non-tradable sector does not increase in same extent, i.e. economy will face a more appreciated real exchange rate.

2.2. TRADABLE PRICES

As discussed in Obstfeld and Taylor (1997), even the studies most favorable to long-run PPP suggest an extremely slow decay rate for international price differentials. Estimated half-lives for

---

\(^{5}\)There are also several other sources of non-stationarity. Increased government consumption, for example, should tend to appreciate the real exchange rate, if the government consumption spending falls more heavily on non-traded goods than does private spending. See Rogoff (1992). However, we have not found any econometric evidence for this phenomena.

\(^{6}\)This is completely true only in a small open economy.

\(^{7}\)See also the discussion in Dornbusch (1989). Necessary conditions for HBS are obviously more acceptable in a long-run.
PPP deviations for most countries and time periods are found to be on the order of four to five years. These estimates appear to imply more sluggishness than one can attribute entirely to nominal rigidities alone.

Engel’s (1999) results suggest that consumers’ prices for tradables goods behave very much in the same way as non-tradables consumer prices. These results are quite puzzling in view of the Harrod-Balassa-Samuelson theory which holds that the relative tradables prices show little long-term variation across countries compared with the variation in relative non-tradables prices. It is certainly true that non-tradable component is important in determination of tradables prices. However, Engel (1999) results again make clear that more than just this must be going on, since prices for relatively tradable goods do not seem to respond any faster to exchange rate movements than do the prices of non-traded goods.

The latest generation of studies on prices and exchange rates has focused more sharply on the issue of markup adjustment as a possible explanation for very slow response of tradable goods prices to exchange movements. Krugman (1987) labeled the phenomenon of exchange rate induced price discrimination in international markets “pricing-to-market”, hereafter PTM. According to the PTM approach international markets for manufacturing goods are sufficiently segmented that producers can, at least over some horizon, tailor the prices they charge to the specific local demand conditions prevailing in different national markets. Although there is a large body of literature suggesting that PTM is indeed important to PPP deviations its implications for the persistence of PPP deviations are not clear. Because the focus of this study is to investigate long-run real exchange rate, PTM is important only if it creates persistent in PPP deviations.

Trade frictions, such as transportation costs, allow even tradables prices to differ within some range without inducing profitable arbitrage. However, this no-arbitrage range is typically assumed to be narrow. Something important must be also happening between the consumer level, where the

---

8 See, for example, Wei and Parsley (1995) or Frankel and Rose (1996).
9 Engel (1999) uses aggregate price indices, but there are also similar findings for highly disaggregate data. See Giovannini (1988) and Engel and Rogers (1996).
10 See Goldberg and Knetter (1997) as a survey.
11 One of the potential weakness of the PTM literature, emphasized by Rogoff (1996), is that it takes the ability to price discriminate as being absolute. This may be the case for some goods, such as automobiles, where differences in national regulatory standards combined with need for warranty service allow firms great leeway to price discriminate across countries. There are, however, a substantial amount of tradable goods which are homogenous in different countries. The findings of Knetter (1993), which show that pricing to market seems to characterize even the most mundane goods, are not in line with this substitutability assumption.
12 Estimates of average transport costs across all tradable goods range between 6 and 10 per cent.
medium-term effect of exchange rates on prices is virtually zero for many goods, and the wholesale level, where price effects tend to be much less than proportional but also significantly greater than zero.\textsuperscript{13} According to Obstfeld and Rogoff (2000), apparent stickiness in terms of domestic currency of the import prices consumers face could result from the pricing practices of domestic importers and distributors, who purchase goods denominated in foreign currency but set retail prices in domestic currency. In that case, importing firms face international prices but the decisions of the ultimate consumers face the retail prices, i.e. not directly international terms of trade prices.

One interesting perspective, which partly combines above arguments, is provided models that emphasize dynamic supply effects. Dixit (1989a and b) show using real option theory that the extent of pass-through depends on the expected changes of nominal exchange rate. To explain the unpredictable responses of trade flows to exchange rate movements, international real option theory stress that investors face sunk entry costs when breaking into foreign markets. Different pricing behavior on different markets now depends on entry and exit decisions of competitive firms. Prerequisites for entry into foreign market are, for example, investment in marketing and distribution network, which are especially important at the consumer level.

\textsuperscript{13} See the results in McCarthy (1999).
3. ECONOMETRICS OF REAL EXCHANGE RATE

As discussed in Juselius (1995) results of empirical estimation of purchasing power parity depends heavily on the econometric method. There are also good reasons to believe that appropriate modeling of prices is crucial to appropriately addressing if PPP condition holds or to determined forces which pull variables away from the equilibrium. We have defined purchasing power parity in Chapter 2 as:

\[ \text{ppp}_t = p_t - p_t^* - s_t \]  \hspace{1cm} (3.1)

In a econometric sense the above definition implicitly assumes that there has to be a homogenous relationship between exchange rate and relative price, i.e. PPP variable should be stationary.\(^{14}\) There is widespread empirical testing of (3.1) for a large number of bilateral exchanges rates showing that the hypothesis of homogenous relationship between exchange rates and relative prices is not an acceptable hypothesis especially based on the recent floating experience. Thus, it is more convenient to define the bilateral PPP for empirical purposes as

\[ \text{opp}_t = p_t - p_t^* - \omega s_t \quad \omega > 0. \]

This supports the choice of cointegration method, which relaxes the assumption of long-run homogeneity between relative prices and exchange rates.

There are also real fundamentals of real exchange rate which may be responsible for introducing a stochastic trend into real exchange rates. This interpretation has received empirical support from researchers who have explicitly modeled the real determinants of real exchange rates.\(^{15}\) However, statistical specification of these studies are sometimes at least dubious. As discussed in Chinn and Johnston (1996), the pitfall of these models is that although they do tend to capture significant Harrod-Balassa-Samuelson links, they often rely on difference specification for bilateral and multilateral rates. Such tests are likely misspecified because the Harrod-Balassa–Samuelson hypothesis is about the relationship between the level of productivity and the level of the real

\(^{14}\text{Generally, the most convincing support for adjustment toward PPP is based on panel unit root research of real exchange rates. See, among others, Papell (1998), Papell and Theodaris (1998), Koedijk, Schotman and Van Dijk (1998), and. Oh (1996).}\)

exchange rate. That is to say, if the series are I(1) then the theory implies that series must be cointegrated and therefore the regression which relies solely on differences will be mispecified from a statistical perspective. It is also important to observe that estimation in the first differences is consistent with the view that there is no meaningful concept of reversion to the productivity-determined equilibrium exchange rate.

A more common shortcoming of real exchange rate models in multivariable case is that prices are usually only implicit variables and statistical properties of the data are not fully recognized. This leads again to use of differenced time series to ensure I(1) property of data, but then all long-run information in the levels of prices and exchange rates have been removed by differencing.\textsuperscript{17} In this paper we follow research tradition created by the paper Johansen and Juselius (1992). All the variables are included in levels. Prices are included explicitly not only implicitly in a real exchange rate term. Possible I(2) trend created by prices will be also analyzed.

The multivariate cointegration technique developed by Johansen (1988, 1991) is used to find steady state relations of real exchange rates. Johansen’s full-system maximum likelihood estimation technique for cointegration testing is based on VAR representation of time series. We consider both the cases of I(1) and I(2) in data generating process. By allowing for a set of conditioning variables, $D_t$, to control for institutional factors, and assuming multivariate normality, the vector autoregressive model is obtained as a tentative statistical model for the data generating process. Let $X_t$ be a $p \times 1$ vector of I(1) variables in the system. Since the Johansen’s basic idea is to distinguish between stationarity by linear combinations and by differencing, we write the model in the following error correction form

$$\Delta X_t = \mu_0 + \mu_1 t + \Gamma_1 \Delta X_{t-1} + \Pi X_{t-1} + \Phi D_t + \varepsilon_t,$$  

(3.2)

where $\varepsilon_t$ is distributed $Niid(0, \Sigma)$ and the parameters $\Theta = \{\Gamma_1, \Pi, \mu_0, \mu_1, \Sigma\}$ are unrestricted. The parameter $\Gamma_1$ defines the short-run adjustment to the changes of the process. The matrix $\Pi$ is estimated by Johansen maximum likelihood procedure. When the process is I(2), it is useful to rewrite model in second order differences.

\textsuperscript{16} See, for example, Hsieh (1982) or DeGregario and Wolf (1994)\textsuperscript{17} Juselius (1999) and Juselius and MacDonald (2000) have, among others, discussed in this theme.
\[ \Delta^2 X_t = \mu_0 + \mu_1 t + \Gamma_1 \Delta^2 X_{t-1} + \Gamma \Delta X_{t-1} + \Pi x_{t-1} + \Phi D_t + \epsilon_t. \] (3.3)

The hypothesis that \( x_t \) is I(2) is formulated as two reduced rank restrictions in Johansen (1991):

\[ \Pi = \alpha \beta' \quad \text{and} \quad \alpha' \Gamma \beta' = \xi \eta', \] (3.4)

where \( \xi \) and \( \eta \) are \((p - r) \times s_1\) matrices (\( s_1 \) is the number of I(1) trends), and the necessary and sufficient conditions for \( x_t \) to be I(2) are that \( \text{rank}(\Pi) = r < p \), \( \text{rank}(\alpha' \Gamma \beta') = s_1 < (p-r) \), and that a further rank condition holds which prevents the variables from being integrated of higher orders.\(^{18}\)

The linear trend coefficient \( \mu_1 \) should restricted to \( sp(\alpha) \), i.e. \( \alpha' \mu_1 = 0 \) as suggested by Rahbek, Konsted and Jörgensen (1998) in order to avoid quadratic trends.

\(^{18}\) More detail discussion, see, Johansen (1992).
4. EMPIRICAL MODELLING OF BALASSA-SAMUELSON HYPOTHESIS

Based on the discussion presented in the Chapter 2, we should find long-run cointegration relations using only the price and productivity variables, if PPP for traded goods held. The idea is to start our analysis with the small Harrod-Balassa-Samuelson model and extend it to full model as a second step of analysis. Obviously, the initial information set should not to be too small to invalidate the identification of relevant cointegration relations. However, we are now testing a specific theoretical relation (the first component of Harrod-Balassa-Samuelson hypothesis) between chosen variables which makes even the result of “no cointegration” interesting. Later, the gradual expansion of the information set facilitates an analysis of the popular ceteris paribus assumption and its importance for empirical analysis of the basic Harrod-Balassa-Samuelson variables. Thus, the first information set analyzed is five variable model

\[ x_t = (s_t, p_t, p_t^*, \text{Pr}_t, \text{Pr}_t^*) \]

where \( s_t \) is the nominal exchange rate, \( p_t \) is the German consumer price index, \( p_t^* \) the U.S. consumer price index, \( \text{Pr}_t \) is the German productivity index, \( \text{Pr}_t^* \) the U.S. productivity index.

The data set used consists of quarterly time series observations from 1975:2 to 1998:1 for Germany and the United States. The main rationale for constructing the sample period to begin in 1975 is to abstract from any transition dynamics associated with the breakdown of Bretton-Woods. All the variables are in logarithmic forms.\(^{19}\)

As seen in the figures 1-9 in the Appendix 1, the observations are strongly time dependent, pointing the need for models based on the adjustment to steady states. Therefore a probability formulation of the whole data set is needed. The unrestricted model was estimated for \( x_t \). The number of lags in VAR was increased until the residuals were Gaussian. Finally, the VAR was specified by installing two lags, which seem to provide a reasonable good approximation of the data generating process.

It is important to find necessary dummy variables because, as Juselius (1995) has pointed out, adding more lags to the model is not a proper remedy for residual autocorrelation, if residual misspecification arises as a consequence of omitting important variables in a dummy vector

\(^{19}\) More detail description of the data is given in Appendix 1.
possibly also leading to heavily overparameterized models. The chosen VAR model needed seasonal dummies and the following two other dummy variables:

\[ D_t = D_{91}, D_{93}, \]

where \( D_{91} \) is a dummy measuring a permanent intervention shock (+1) in 1991:2 and 1991:3 and \( D_{93} \) a permanent intervention shock (+1) in 1993:1. Thus, we have used two dummies especially designed for German reunification.\(^{20}\) Both dummies are supported by the data set with the t-values 3.5 for \( D_{91} \) and 4.08 for \( D_{93} \). Since all empirical models are inherently approximations of the actual data generating process, the question is whether our model is a satisfactory close approximation. To investigate this issue we test the stochastic specification regarding residual correlation, heteroscedasticity and normality. Test statistics are represented in Table 4.1. below.

<table>
<thead>
<tr>
<th>Table 4.1. Misspecification tests.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual autocorr.</td>
</tr>
<tr>
<td>Normality</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Univariate tests</td>
</tr>
<tr>
<td>ARCH(2)</td>
</tr>
<tr>
<td>Normality</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Ex. Kurtosis</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
</tbody>
</table>

Neither of multivariate tests are significant. This is an advantage for the model because the VAR model is based on the assumption of multivariate normal disturbances, i.e. residuals should behave approximately as a multivariate normal process. Univariate tests test for normality of the individual residuals can be rejected as a result of skewness (third moment) or excess kurtosis (fourth moment). Since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis we reported the third and forth moment around the mean.\(^{21}\) Because there does not seem to be a serious skewness problem, we conclude that the normality conditions are satisfactory.

\(^{20}\) Also the step dummy, 0 before reunification and 1 after, was examined but it did not work a satisfactory way.

\(^{21}\) See the discussion in Gonzalo (1994).
Selecting proper critical values for testing cointegration rank depends on the nature of the deterministic components and the order of integration of the data. In the following analysis all test statistics have been calculated under the assumption that data contains linear but not quadratic trends. *A priori*, the differences between components in terms of persistence are due to the order of their stochastic trends rather than differences in the deterministic part.\(^\text{22}\) The linear trend may, of course, have zero coefficients in certain directions. However, whether a trend is present does not affect the asymptotic properties of tests and estimators in chosen model.\(^\text{23}\) In addition, because our data vector $x_t$ might be of second order instead of first order nonstationary the asymptotic distributions based on I(1) assumption might be violated. For example, asymptotic distributions of conventional tests used to find the correct inference on the number of cointegrating vectors, such as Trace and Max test, might be misleading if the order of integration is two.\(^\text{24}\)

We discuss the choice of rank based on the additional information given by the $p \times k = 10$ roots of companion matrix.\(^\text{25}\) The number of unit roots in the characteristic polynomial is $s_1 + 2s_2$, where $s_1$ and $s_2$ are the number of I(1) and I(2) components respectively. The intuition is that the additional $s_2$ unit root belong to $\Delta x_t$, hence, to the $\Gamma$ matrix in (3.3). Therefore, the roots of the characteristic polynomial contain information on the unit roots associated with both $\Gamma$ and $\Pi$, whereas the standard I(1) trace test only contains information on unit roots in the $\Pi$ matrix. The order of integration and cointegration can be formally tested in the I(2) model using the likelihood procedure. Johansen (1995) derived a LR test for the determination of $s_1$ conditional on chosen $r$. Paruolo (1996) extended the test procedure to the joint determination of $(r, s_1)$ and Rahbek, Kongsted and Jörgensen (1998) derive the nonstandard asymptotic distributions for trend stationary in the I(2) model. Our results gave some evidence for two stochastic I(2) trend.\(^\text{26}\) Especially, even a price differential seems to be I(2). The results also suggested that the reduced rank of $\Pi$ was two, i.e. $r = 2$. The real exchange transformation itself might be an I(1) trend. Thus, we conclude that $r = 2$, $s_1 = 1$ and $s_2 = 2$.\(^\text{27}\)

\(^{22}\) As simulations of Doornik, Hendry and Nielsen (1998) have shown, even if the DGP did not include the trend its adoption into cointegration space would only have a low cost.  
\(^{23}\) A property of asymptotic similarity, see Rahbek et. al (1998)  
\(^{24}\) Jörgensen (1998) demonstrates the low power of the trace tests in I(2) or near I(2) models.  
\(^{25}\) The discussion about characteristic roots and companion matrix see, for example, Kongsted (1998).  
\(^{26}\) Complete I(2) analysis is available by request from the author.  
\(^{27}\) However, we have to admit that the econometric evidence of the fifth unit root was not empirically robust.
More precise inference in the I(2) model would require distributional results not yet available. To circumvent this problem we use a transformation of the I(2) model instead, such that inference can proceed in the framework of the statistically well-understood I(1) model. In order to use PPP in the transformation vector we should first investigate a possible long-run homogeneity between variables included in the transformation. The result concerning the variables in CI(2,1) relation, are quite satisfactory. This hypothesis is accepted at five percent significance level with the likelihood ratio test statistic 4.91 $\chi^2(3)$. It is also possible to test whether the long-run homogeneity assumption can be imposed in all cointegration relations. The likelihood ratio test statistic 28.91 is asymptotically distributed as $\chi^2(4)$. Thus, and not surprisingly, the hypothesis concerning the overall long-run homogeneity between nominal exchange rate and price differential in the cointegration space is clearly rejected. These results seem, however, to suggest the relation between price differential and nominal exchange rate. We can conclude that it is likely that $p_t - p_t^* - s_t$ is CI(2,1), but not with the unitary coefficient in all cointegration relations. These contradictory findings are probably attributable to the borderline integration of nominal exchange rate between I(2) and I(1) trend. However, if the model is estimated by using this PPP transformation, there is no evidence for I(2) in the data. This is a very promising result and we will analyze this relation more closely.

4.1. EMPIRICAL ANALYSIS OF THE TRANSFORMED I(1) MODEL

The empirical analysis of the first I(1) data set will be based on the PPP transformed vector:

$$(ppp_t, Pr_{o_t}, Pr_{o_t}^*, \Delta p_t, \Delta p_t^*)$$

where $ppp = (p_t - p_t^* - s_t)$. If the PPP restriction (1,-1,-1) had been acceptable in all cointegrating relations in the I(2) model, the VAR model analysis of this transformation would have been empirically equivalent and no long-run information would have been lost by the transformation.\textsuperscript{28} However, based on the results reported in the previous chapter, the joint restrictions were not data consistent in I(2) data set and some information is lost by using transformed vector. Thus, we have to admit that PPP transformation is a little problematic when implied restrictions are not statistically

\textsuperscript{28} See the discussion in Juselius and Toro (1999) and Juselius and MacDonald (2000).
acceptable. Another solution would have been to remove I(2) by differencing. This would have resulted a greater information loss and would be inconsistent with economic theory. Since the likelihood function of the real exchange rate transformed model is now changed as compared to the analysis in previous section, we have recalculated the misspecification tests. The stochastic specification regarding residual correlation, heteroscedasticity and normality indicates that the model can be considered a satisfactory description of the data generating process.

Under the assumption that the model is a well defined statistical process, we next determine the number of cointegration vectors. The status of deterministic terms like constant or trend has been determined testing the joint hypothesis of both the rank order and the deterministic components as suggested in Johansen (1992). Trends in the cointegration relations are supported by this test procedure. The hypothesis of the exclusion of trend in a cointegration space is also rejected \( \chi^2(5) = 14.48 \) with the p-value 0.01.

Although test statistics accept the modelling of the vector where a linear trend is included in cointegration space, this choice is partly problematic in this data vector. The linear trend should clearly be present in the I(2) model since prices have got a linear trend in them, but not necessarily in this PPP transformed I(1) model. If the trend is excluded in the cointegration space, however, the largest estimated eigenvalue of the companion matrix is outside the unit circle. Under the assumption of the cointegrated VAR model, the eigenvalues should be inside the unit circle or equal to unity. Eigenvalues outside the unit circle correspond to explosive processes and the model we have chosen is not an adequate description of such data. Thus, we have chosen the model with the linear trend in cointegration space although admitting this choice is partly problematic.

It is common practice to use trace test to determinate the number of cointegration vectors. However, the trace test has low power against near cointegration alternatives especially in a small sample. Beside this, it is not straightforward to use the tabulated critical values for the trace test since the distribution of these are simulated under the assumption no weak exogeneity. Because we have found evidence on this phenomenon no conclusion will be solely based on trace test result but, in addition, the inference of numbers of cointegration vectors will be defined detecting the roots of

\[ \chi^2(5) = 14.48 \]
characteristic polynomial. This test procedures support the choice of \( r = 2 \) which is in line with the pervious findings in I(2) model. The signs of I(2) components have now disappeared, i.e. the roots of the characteristic polynomial are now consistent with I(1) model.\(^{32}\)

To investigate the time series properties of the individual variables and their status in the system, three different tests are reported in Table 4.2. The test of stationarity indicates that none of the variables can be considered stationary. Based on the test for long-run exclusion all variables are found to be significant for the long-run structure.\(^{33}\) The test of long run weak exogeneity investigates the absence of long run levels feedback. This test shows that PPP and German productivity can both be considered weakly exogenous for \( \beta \). The test of both being jointly weakly exogenous is accepted based on \( \chi^2(4) = 0.84 \). The data strongly supports the hypothesis that the PPP term is not an adjusting variable in a cointegration space. This finding is not conditional on the number of cointegration vector. Thus, the test of weak exogeneity provide evidence not in favor of our theory for determination of the real exchange rate.

### Table 4.2. Properties of system variables

<table>
<thead>
<tr>
<th></th>
<th>Chi(v)</th>
<th>ppp</th>
<th>dp</th>
<th>dp*</th>
<th>Pro</th>
<th>Pro*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationarity</td>
<td>5.99</td>
<td>36.83</td>
<td>22.20</td>
<td>45.66</td>
<td>23.13</td>
<td>30.86</td>
</tr>
<tr>
<td>Exclusion</td>
<td>9.49</td>
<td>30.66</td>
<td>25.11</td>
<td>23.83</td>
<td>35.97</td>
<td>35.83</td>
</tr>
<tr>
<td>Weak exogen</td>
<td>5.99</td>
<td>0.46</td>
<td>25.16</td>
<td>49.29</td>
<td>0.47</td>
<td>14.93</td>
</tr>
</tbody>
</table>

Although the test of weak exogeneity suggests the Harrod-Balassa-Samuelson theory is not an adequate theory to explain the trending behavior of real exchange rate, we have tested whether there is a long-run equilibrium among the variables determined by this theory. We cannot find any evidence for the Harrod-Balassa-Samuelson effect using this data vector. Thus we can conclude that the real exchange rate is not an adjusting variable and there is no long-run relation between productivity variables and real exchange rate, i.e. productivity differential does not seem to be able to explain the nonstationary behavior of real exchange rate.

---

\(^{32}\) Five largest roots of the companion matrix are (0.95; 0.95; 0.88; 0.47; 0.47) in unrestricted model and (1.00; 1.00; 1.00; 0.63; 0.47) in restricted \((r=2)\) model.

\(^{33}\) Although not reported here the test for long-run exclusion accepts the linear trend in cointegration space.
5. STOCHASTIC FORMULATION OF THE REAL EXCHANGE MODEL

There has recently been an increased interest in the small sample properties of cointegration tests. Since the null hypothesis of a unit root is not necessarily reasonable from an economic point of view, the low power is a serious problem. Economic theory suggests often a priori hypothesis for the number of independent trends. This is a strong argument for building the choice of r on economic theory as well as on statistical information in the data. In this chapter we will give a stochastic formulation of the real exchange rate model.

We will now analyze the econometric consequences of theoretical real exchange rate models. PPP and other relations will be given a stochastic formulation based on the decomposition of the data into once or twice cumulated shocks and a stationary component. In order to illustrate the main target of this chapter we first examine the Harrod-Balassa-Samuelson real exchange rate model in a world with no market rigidities, no trade barriers, no restrictions on capital movements, no transportation costs and fully integrated goods markets. Based on the results presented in a previous chapter we assume that prices are I(2) in both countries. This is again only the specific assumption for this time period not a general assumption on the time series properties of price indices. The data generating process (DGP) could then be represented as:

\[
\begin{bmatrix}
  p_t \\
  p_t^* \\
  s_t \\
  Pr_o \\
  Pr_o^* \\
  \Delta p_t \\
  \Delta p_t^*
\end{bmatrix} = \begin{bmatrix}
  1 \\
  1 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix} \sum \Sigma \mu_{1i} + \begin{bmatrix}
  1 & 1 \\
  1 & 1 \\
  0 & 0 \\
  0 & 1 \\
  1 & 1 \\
  1 & 1
\end{bmatrix} \begin{bmatrix}
  \sum \Sigma \mu_{1i} \\
  \sum \Sigma \mu_{2i}
\end{bmatrix} + X_0
\]

where \( p_t \) is home country price index, \( p_t^* \) foreign country price index, \( s_t \) is nominal exchange rate, \( Pr_o \) home country productivity, \( Pr_o^* \) foreign country productivity. \( \sum \Sigma \) indicates twice cumulated shock, \( \sum \) once cumulated shock and \( X_0 \) includes a stationary component and possibly also a deterministic trend. Under the assumption of flexible prices and strong market integration between

---

34 See, for example, Johansen (1998).
35 We later discuss the possible I(2) property of nominal exchange rate.
Germany (home country) and the United States (foreign country) we would except one common nominal price trend, $\mu_i$, in the data. The other shock of our model, $\sum \mu_i$, is defined as a real shock.

The price differential between economies is a stationary relation due to the strong market integration. Thus, there is a short run price homogeneity between $\Delta p_t$ and $\Delta p_t^*$ which makes $p_t$ and $p_t^*$ CI(2,0) with cointegration vector (1,1). PPP is a stationary relation between price differential and stationary nominal exchange rate. It is also plausible to assume in this complete market world that there are rapid technology transfers between countries i.e. there are identical technologies in both countries. This creates a stationarity relation between productivity variables. Thus, the number of cointegrated vectors is three (one extra stationary vector created by the stationary nominal exchange rate variable). There is also one I(2) trend and one I(1) trend.

5.1. DECOMPOSITION OF THE PRICES

There is no empirical support in our data for a stationary relation between prices implying a lack of market integration between two goods markets. This is in line with the discussion in Goldberg and Knetter (1997). According to this research there appears to be price discrimination between European and American markets, dictated by distinct competitive conditions in those markets. In fact, our finding is $p_t - p_t^* \sim I(2)$ and there seems to be no short run price homogeneity from which follows $\Delta p_t - \Delta p_t^* \sim I(1)$. Because even the inflation differential seems to be a non-stationary variable there might be a structural change in inflation process especially in the U.S.A as seen in the Figure 8 (Appendix 1). The nonstationary condition of the inflation rate differential variable can be easily seen in Figure6.1. 36

36 See also the results presented in Table 6.3.
Relaxing the assumptions of perfect market integration and flexible prices, thereby allowing for different trends in nominal prices would decrease the amount of cointegrated vectors between two countries. The trend components of two price indices can be written as:

\[ p_t = a_{1i} \sum \sum \mu_{li} + a_{12} \sum \sum \mu_{2i} + b_{1i} \sum \mu_{li} + b_{12} \sum \mu_{2i} + X_0 \]

\[ p_t^* = a_{2i} \sum \sum \mu_{li} + a_{22} \sum \sum \mu_{2i} + b_{2i} \sum \mu_{li} + b_{22} \sum \mu_{2i} + X_0 , \]

where \( a_{ii} \neq 0 \) (i = 1,2) indicating that prices are I(2). Based on the results in I(2) analysis, prices seem to include two I(2) trends. Following the above statements we define them as nominal and real shocks. In our data set coefficients (a and b) are not necessary zero or one as in an illustrative example. We also assume, \( a_{11} + a_{12} = a_{21} + a_{22} \), which would be consistent with the different stochastic price trends, excluding the possibility of any cointegration between two price indices.
5.2 DECOMPOSITION OF THE NOMINAL EXCHANGE RATE

In order to discuss the real exchange rate we also must define the statistical properties of nominal exchange rate. The results of I(2) analysis show that the nominal exchange rate might be affected by one I(2) trend. Additionally, based on good theoretical arguments, \textit{a priori}, nominal exchange rate and price differential should share a common trend. Thus, we conclude that the nominal exchange rate is partly I(2) variable although admitting that the evidence is not empirically robust.

![Figure 6.2. Nominal exchange rate and price differential.](image)

The large deviations from the long-run price trend of nominal exchange rate in the chosen data are shown in Figure 6.2 above. This together with the econometric evidence of the I(1) analysis suggests that the nominal exchange rate has been also strongly affected by an additional stochastic I(1) trend, $\sum \mu_{3i}$, which does not seem to affect the price differential.

Juselius and MacDonald (2000) identified this extra stochastic trend as a “reserve currency” trend. The role of the U.S dollar as a reserve currency is likely to have resulted in permanent shocks to the nominal exchange rate that are unrelated to pure price shocks. However, we have modeled the third stochastic shock explicitly by using a real oil price as a proxy for permanent shock in the U.S dollar. Following the results presented in the next chapter higher oil price leads to an appreciation of the U.S. dollar in a long run. If the shock affects the nominal exchange rate without affecting the price differential we should find a nonstationary real exchange rate if this shock is not included at least
implicitly in the model. Thus, we should rationalize the oil price shock which appreciates the nominal US dollar relative to the German mark and creates a lasting price differential between price indices in order to explain a nonstationary real exchange rate in our data set.

The positive relationship between U.S. dollar and oil price is partly problematic because, being a major importer of crude oil, higher oil price worsen the U.S. terms of trade. This should depreciate the U.S. dollar not appreciate it relative to the German mark because oil price changes on the United States terms of trade affect more negatively than German terms of trade. Interestingly, Amano and van Norden (1995), present evidence of a similar effect for Canada, where higher oil price leads to a weaker Canadian dollar relative to the U.S. dollar despite the fact that Canada is a substantial exporter of oil and the U.S. a net importer of crude oil.

However, this finding is not entirely counterintuitive. First, the save haven effect might appreciate the U.S. dollar, because of increased uncertainty in a world economy created by the oil price shock. Secondly, according to Krugman (1983), the effect of the price of oil depends on whether the burden to a country to country’s balance of payments created by higher oil imports is greater or less than the improvement due to OPEC investment and imports. Thus, it is important to consider the effect of oil price shocks on exchange rates in a multi country framework. In a three-country world (Germany, the United States and OPEC) higher oil price will transfer wealth from oil importers (Germany and the United States) to the oil exporters. While American and German current accounts are thus worsened, however, there is an improvement in capital accounts as OPEC invests its trade surplus in dollars and marks. Whether the net effect is favorable or unfavorable for the dollar depends on whether OPEC investment in dollars is more or less than America’s share of the industrial world’s current account deficit. All in all, the case of oil price movements offers an interesting example of possible conflict between an asset market and a goods market view of the exchange rate.

---

37 Crude oil is a complement to home goods in a medium run. In the long run Germany has decreased its dependence on oil more rapidly than the United States. See Backus and Crucini (1998).
38 See the discussion in Backus and Crucini (1998).
39 In fact, this explanation is very similar with the “reserve currency” explanation discussed in Juselius and MacDonald (2000).
40 The implicit assumption is that OPEC spending lags behind income. Thus, over time the balance of payments effect of higher oil prices depend upon its preferences for goods, i.e. trade flows are more important in the long-run. This might also have a positive effect on dollar value based on the discussion in Amano and Van Norden (1998).
41 We have here excluded the market expectations of exchange rate changes. If we adopt the hypothesis of rational expectations a trade flow effect can dominate the financial one even from the start.
Following a real option approach discussed in Chapter 2, the uncertainty of the permanence of the oil shock which causes relative price changes through nominal exchange rate changes will widen the range within which the price differentials can fluctuate. If the uncertainty as to the permanence of the oil shock causing nominal exchange rate changes is high, the risk adjusted profit of arbitrage of goods is not necessarily high enough for arbitrageurs to engage in. Then, this increases the time before arbitrage condition commences due to the option value of waiting.

Although we do not argue that oil prices are immune to the laws of supply and demand it seems to be quite a reasonable argument that oil prices are heavily dependent on the stability of cartels. The most important cartel for an oil price is obviously the OPEC cartel. However, the history of the rise and fall of oil prices is also highly suggestive of some sort multiple equilibrium story. The original surge in oil prices came suddenly and unexpectedly, something what would be expected if events moved the market from one equilibrium into another. The collapse of oil prices in 1986 also came with dramatic suddenness, again suggestive of a collapse of one equilibrium and establishment to another.42 This possible multiple equilibrium nature of the oil price makes oil price a very unpredictable variable.43

Following the above discussion the nominal exchange rate is now defined as:

\[ s_t = a_{32} \sum \sum \mu_{1i} + b_{31} \sum \mu_{2i} + a_{32} \sum \mu_{2i} + b_{33} \sum \mu_{3i} + X_0 \]

Again, if \( a_{32} \neq 0 \), the nominal exchange rate is I(2). There is only one I(2) trend in nominal exchange rate decomposition based on the results of I(2) analysis discussed in Chapter 4. This is not, however the basic reason for long-run deviations from price differential trend, because nominal exchange rate and price differentials are cointegrated (CI(2,1)). In order to explain these large and persistence deviations from PPP, there should exist also the third once cumulated shock in a nominal exchange rate decomposition as discussed above.

---

42 This multiple equilibrium idea is discussed in the article “Energy crisis revisited” available at official Paul Krugman web-site.
43 The possible multiply equilibrium nature of oil price would change the real option analysis by changing the time series representation of real exchange rate. Dixit (1989), Krugman (1989), among others, have assumed Brownian motion representation but oil price may introduce a Poisson jump process, i.e. exchange rate would now follow a mixed Brownian motion-Poisson jump process. However, we leave this subject for further research.
5.3. DECOMPOSITION OF THE FULL MODEL

The above discussion is summarized in the following matrix:

\[
\begin{bmatrix}
    p_t \\
p^*_t \\
s_t \\
Pr\ o^*_t \\
Pr\ o_t \\
Oilp_t \\
\Delta p_t \\
\Delta p^*_t \\
\Delta s_t
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
0 & a_{32} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\sum \sum \mu_{1i} \\
\sum \sum \mu_{2i} \\
\Sigma \mu_{1i} \\
\Sigma \mu_{2i} \\
\Sigma \mu_{3i}
\end{bmatrix} + \begin{bmatrix}
b_{11} & b_{12} & 0 \\
b_{21} & b_{22} & 0 \\
b_{31} & b_{32} & b_{33} \\
0 & b_{42} & 0 \\
0 & b_{52} & 0 \\
0 & 0 & b_{63} \\
a_{11} & a_{12} & 0 \\
a_{21} & a_{22} & 0 \\
0 & a_{32} & 0
\end{bmatrix} X_0
\]

The cointegration properties of the data can be discussed using the above matrix. If \( a_{11} = a_{21} \) and \( a_{12} - a_{22} = a_{32} \), then \( p_t \) and \( p^*_t \) are cointegrated with \( s_t \), \( Pr \ o^*_t \), \( Pr \ o_t \), \( Oilp_t \), \( \Delta p_t \), \( \Delta p^*_t \), and \( \Delta s_t \). The number of cointegration vectors in the full model should be at least three. (\( r = 2 \) in a previous analysis).
6. REAL EXCHANGE RATE MODEL WITH THE OIL PRICE

In this chapter we will discuss estimation results based on the data vector including real exchange rate, two productivity variables, price differentials and a linear trend as in a previous estimation, but now also real oil price. We will show the importance of real oil price for determination of stationary PPP relation. The dummy vector now includes also a special dummy designed for the Gulf War. The chosen VAR model needed the following three dummy variables:

\[ D_t = D_{o91}, D_{91}, D_{93}, \]

where D91 and D93 are the same intervention dummies as those discussed in Chapter 4. Do91 is a Gulf War dummy measuring a transitory shock defined as plus one in 1990:3, minus one 1991:1 and zero otherwise. Despite the inclusion of this dummy, the residual of oil price variable is not normally distributed and also the multivariate normality assumption is violated. This is not surprising since the real oil price was especially chosen to explain the variation in real exchange rate but not vice versa, meaning that the selected variable set is probably not sufficient to account for the variation in the real oil. The real oil price variable might be weakly exogenous for the long-run parameters of interest, which would make the deviation from normality less important (see test results in Table 6.2.). Residual normality is also mainly rejected due to excess kurtosis. Because cointegration results appear robust to excess kurtosis, we have ignored this normality problem. The results of the misspecification tests are reported in Table 6.1.

Table 6.1. Misspecification tests and characteristic roots

<table>
<thead>
<tr>
<th>Multivariate tests</th>
<th>CHISQ(25) = 44.95</th>
<th>CHISQ(25) = 40.18</th>
<th>CHISQ(10) = 27.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual autocorr.</td>
<td>p-val = 0.16</td>
<td>p-val = 0.29</td>
<td>p-val = 0.01</td>
</tr>
<tr>
<td>LM (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM (4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>0.60</td>
<td>0.20</td>
<td>5.78</td>
</tr>
<tr>
<td>Normality</td>
<td>3.34</td>
<td>4.09</td>
<td>0.69</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.38</td>
<td>0.40</td>
<td>-0.05</td>
</tr>
<tr>
<td>Ex. Kurtosis</td>
<td>0.54</td>
<td>0.70</td>
<td>0.10</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.36</td>
<td>0.76</td>
<td>0.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Univariate tests</th>
<th>0.19</th>
<th>0.19</th>
<th>1.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>dppp</td>
<td>2.03</td>
<td>2.75</td>
<td>23.74</td>
</tr>
<tr>
<td>ddp</td>
<td>0.34</td>
<td>-0.01</td>
<td>-0.32</td>
</tr>
<tr>
<td>ddp*</td>
<td>-0.68</td>
<td>-0.39</td>
<td>3.35</td>
</tr>
<tr>
<td>dPro</td>
<td>0.43</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>dPro*</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>dOilpr</td>
<td>0.34</td>
<td>0.79</td>
<td>0.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Six largest roots of the process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestricted</td>
</tr>
<tr>
<td>r = 4</td>
</tr>
<tr>
<td>r = 3</td>
</tr>
</tbody>
</table>
The inference on the number of cointegrated vectors was again made on the basis of the roots of characteristic polynomial. This supports the choice of \( r = 3 \), which is also consistent with theoretical discussion in a previous chapter. As a sensitivity check, the roots under the choice \( r = 4 \) are also reported in Table 6.1. As a check of the properties of the system variables tests for stationarity, long-run exclusion and weak exogeneity is reported in Table 6.2.

Table 6.2. Properties of the variables if oil price is included in the data vector.

<table>
<thead>
<tr>
<th>Property</th>
<th>Chi(v)</th>
<th>ppp</th>
<th>dp</th>
<th>dp*</th>
<th>Pro</th>
<th>Pro*</th>
<th>Oilpr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exclusion</td>
<td>7.81</td>
<td>31.52</td>
<td>22.56</td>
<td>44.75</td>
<td>21.73</td>
<td>27.48</td>
<td>10.68</td>
</tr>
<tr>
<td>Weak exogeneity</td>
<td>7.81</td>
<td>11.00</td>
<td>26.11</td>
<td>57.61</td>
<td>1.33</td>
<td>17.80</td>
<td>7.93</td>
</tr>
</tbody>
</table>

All variables were found to be significant for the long run structure and the weak exogeneity results show that German productivity and possibly oil price can be considered weakly exogenous for the long term parameter \( \beta \). However, the most interesting result is that PPP term is not weakly exogenous if the real oil price is included in the data vector.

6.1. STRUCTURAL HYPOTHESES ABOUT THE COINTEGRATION SPACE

The purpose of this section is to demonstrate that the some of the economic hypotheses discussed in Chapter 2 and Chapter 6 can be given a precise statistical formulation, and therefore, can be validly tested. In the ideal case the specification of the matrices should define a identifying structure with all freely estimated coefficients statistically significant and economically interpretable. Hypothesis 1-6 are of the form \( \beta = \{H, \phi_1, \psi_1, \psi_2\}_i=1..6 \); that is, they test whether a single restricted relation is in the cointegration space, leaving the other two relations unrestricted. The results are given in Table 6.3.

44 Again, linear trend in cointegration space was supported by this test procedure.
45 For the derivation of the test procedures, see Johansen and Juselius (1992).
Table 6.3. Tests on the cointegration vectors.

<table>
<thead>
<tr>
<th></th>
<th>ppp</th>
<th>dp</th>
<th>dp*</th>
<th>Pro</th>
<th>Pro*</th>
<th>Oilpr</th>
<th>Trend</th>
<th>Chi(v)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10.89</td>
<td>0.02</td>
</tr>
<tr>
<td>H2</td>
<td>0</td>
<td>1</td>
<td>-0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8.66</td>
<td>0.03</td>
</tr>
<tr>
<td>H3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>16.95</td>
<td>0.00</td>
</tr>
<tr>
<td>H4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-0.84</td>
<td>0</td>
<td>0</td>
<td>12.95</td>
<td>0.00</td>
</tr>
<tr>
<td>H5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.003</td>
<td>0.00</td>
</tr>
<tr>
<td>H6</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.53</td>
<td>0</td>
<td>12.88</td>
<td>0.00</td>
</tr>
<tr>
<td>H7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>6.67</td>
<td>-6.67</td>
<td>0</td>
<td>0</td>
<td>13.31</td>
<td>0.00</td>
</tr>
<tr>
<td>H8</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>12.73</td>
<td>-11.75</td>
<td>0</td>
<td>0</td>
<td>11.63</td>
<td>0.00</td>
</tr>
<tr>
<td>H9</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.27</td>
<td>0.01</td>
<td>3.71</td>
<td>0.16</td>
</tr>
<tr>
<td>H10</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-17.90</td>
<td>17.90</td>
<td>2.04</td>
<td>0</td>
<td>7.96</td>
<td>0.02</td>
</tr>
<tr>
<td>H11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-9.27</td>
<td>11.24</td>
<td>2.26</td>
<td>0</td>
<td>0.57</td>
<td>0.45</td>
</tr>
<tr>
<td>H12</td>
<td>-0.03</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.02</td>
<td>0</td>
<td>0.06</td>
<td>0.97</td>
</tr>
<tr>
<td>H13</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-0.078</td>
<td>0</td>
<td>0</td>
<td>-0.001</td>
<td>0.36</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Hypotheses H(1) and H(2) are related to the inflation differential. A stationary relationship between inflation rates is rejected although hypothesis two is almost accepted at a 5% significance level.

Hypotheses H(3) and H(4) are related to long-run relationship between productivity variables in Germany and the United States. We have not found stationary relation between productivity variables in our data, which includes 23 years. Thus, it seems to be take a relatively long time period before the assumption of similar technology is accepted even in the case of two industrialized countries.

Hypotheses H5 and H6 are related to the PPP variable. It is a common habit to model possible long-run deviations from PPP as a linear trend, i.e. if long-run deviations are due to the Harrod-Balassa-Samuelson hypothesis the linear trend represents the productivity differential. Based on the results concerning hypothesis five we have not found evidence for this relation. H6 tests whether there is a stationary relation between PPP and real oil price. Amano and Van Norden (1999) has found this relation using the US real effective real exchange rate but here we have not been able to find stationary relation between PPP and the real oil price.

Hypotheses H7-H11 test the PPP relation in multivariable environment. These are all tests of the type, $\beta = \{H, \phi_i, \psi_i\} i=7...11$, where $(H, \phi_i)'$ is given by the hypothetical vector related hypotheses 7-11 in Table 6.3. and $\psi = (\psi_{i1}, \psi_{i2})$ is a matrix of unrestricted coefficients. Hypotheses H7 and H8 clearly reject the standard Harrod-Balassa-Samuelson hypothesis. H10 introduces real oil price in a
multivariable environment. The most interesting result is the hypothesis 11. We are able to find a traditional Harrod-Balassa-Samuelson result if the oil price is included in the data vector between the German mark and the U.S. dollar with the test statistics $\chi^2(1) = 0.57$.\(^{46}\) The result suggests that oil prices may have been an important source of persistent real exchange rate shocks. However, before drawing any conclusions we have to first recognize the full cointegration space and also do a stability analysis.

H12 and H13 identified the other two cointegration vectors. H12 defines U.S inflation using a real exchange rate and oil price. H13 is probably best understood as an inflation differential between Germany and the U.S.A. The linear trend might indicate the increased anti-inflation credibility of the FED which has decreased the inflation differential between the currencies. Undoubtedly, the economic rationality of these two relations is a far from perfect. Thus, these relations give us reason to suspect that other variables than those “real economy” variables included in the analysis probably play an important role, and only with a much bilier system it is possible obtain a model that satisfies both statistical and economic interpretability.

Using the above results it is now time to move a structural formulation of the full cointegration space expressed as the following joint hypothesis, $\beta = \{H_1\phi_1, H_2\phi_2, H_3\phi_3\}$, where the design matrix $H_i$ defines the assumed structural representation.\(^{47}\) The joint hypothesis about the long-run structure was formulated by the following design matrices:

\[
H_1 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \quad H_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
\end{bmatrix} \quad H_2 = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\(^{46}\) Typically the oil price effect has been analyzed using multilateral exchange rates, as an example, see MacDonald (1997) or Amano and Van Norden (1998).

\(^{47}\) This test procedure is discussed in Johansen and Juselius (1994).
These matrixes combine H11, H12 and H13 and are together safely accepted with a test statistic 2.39 asymptotically distributed as $\chi^2(5)$ and p-value of 0.79. The estimates of the unrestricted $\beta_j$ coefficients and their asymptotic standard errors are given in Table 6.4.

Table 6.4. Cointegration vectors.

<table>
<thead>
<tr>
<th>Vector</th>
<th>Coefficient</th>
<th>Stand. error</th>
<th>PPP</th>
<th>Dp</th>
<th>Dp*</th>
<th>Pro</th>
<th>Pro*</th>
<th>Oilpr</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector 1</td>
<td>1</td>
<td>-1.46</td>
<td>-10.14</td>
<td>11.84</td>
<td>2.14</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1.42</td>
<td></td>
<td></td>
<td>1.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td>Vector 2</td>
<td>-0.027</td>
<td>0.002</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.002</td>
</tr>
<tr>
<td>Vector 3</td>
<td>0</td>
<td>0.02</td>
<td>-1</td>
<td>-0.092</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The results in the above table show that all freely estimated coefficients are significant, implying that the suggested structure is empirically identified. All three cointegration relation are presented in Appendix 2.

It might be also informative to examine the structure of the $\alpha$ coefficients. The short run adjustment to the PPP relation takes place primarily in inflation variables, but to some extent also in the real oil price variable. The real oil price is a borderline finding with the t-value 2.15.\textsuperscript{48} Although it is possible that OPEC consider the effects of oil price changes, oil price probably should be a weakly exogenous variable in this cointegration space. The PPP, inflation and U.S productivity variable are significant in the U.S inflation equation. The short run adjustment to the third cointegration vector, defined as inflation differential equation, is mainly German and U.S. inflation.

\textsuperscript{48} This finding partly explains why we did not accept a weak exogeneity of real oil price in test procedure reported in the Table 6.2.
The stability of the results has been checked by a recursive stability test suggested Hansen and Johansen (1993).

Figure 6.1. Recursive estimates (base year 1990).

Figure 6.1. shows recursively calculated test statistics for the test of a constant cointegration space. There are some problems in vector Beta Z (upper line) which pictures the actual deviations as a function of all short run dynamics including seasonals and other dummies. Beta R is corrected for the short-run effects and seems to be more stable. This supports the parameter constancy for the period we investigate.49

Altogether, we find that the results are interpretable and provide new insight into dynamics of the highly complex long-run adjustment process of a German mark/U.S. dollar real exchange rate. Generally results shows that the real exchange rate and the goods market are related in such a way that the PPP or the Harrod-Balassa-Samuelson effect alone cannot explain this complex structure.

49 We have tested stability of cointegration space using several base years. These findings also support the parameter constancy.
7. CONCLUSIONS

There is a considerable amount of evidence that the purchasing power parity does not hold during the recent float between German mark and the U.S. dollar. In the short run, it may be due to the price stickiness. But in the long run price stickiness cannot possibly matter and, hence, deviations from purchasing power parity should be accounted for by real factors. However, it is a common finding in the empirical real exchange rate literature that exchange rates will often and persistently wander away from empirical estimates of their long run equilibrium values, whether these are determined by a PPP relationship or a more elaborate estimation of fundamental equilibrium levels that takes into account shifts in the real factors.

Number of studies have furnished fairly persuasive evidence, that deviations from PPP derive in large part from differences in relative traded goods prices across countries. Recently, there has been arguments in the literature that the failure of PPP to hold even for traded goods may be largely an U.S. dollar phenomenon. Engel (1999) has shown that the movements in the relative price of traded goods are important for the U.S. real exchange rate. Canzoneri, Cumby and Diba (1999) use panel cointegration methods to examine the Harrod-Balassa-Samuelson effect. They argue that the problems with the Harrod-Balassa-Samuelson hypothesis lie in the failure of PPP to explain traded goods prices, especially for the U.S dollar.

We have discussed oil price as a possible source of the failure of PPP for traded goods, when the U.S. dollar real exchange rate is considered. Trade frictions, such as transportation costs, allow even tradables prices to differ within some range without inducing profitable arbitrage. When arbitrage is immediate transport costs strictly delimit the range of price fluctuations. However, in the presence of sunk cost of arbitrage uncertainty as to the permanence of the shocks causing relative price changes will widen the range within which price differentials can fluctuate and similarly increase the time before arbitrage commences. We have introduced a real option investment theory, inspired by Dixit (1989a and b), to discuss persistent deviations from the PPP. Oil price is also shown to be the important source of uncertainty, which will appreciate the US dollar without the complete price arbitrage of tradable goods.

The first model was I(2) model. We have found no evidence for stationary real exchange rate. In fact, the empirical evidence for the cointegration relation between price differential and the nominal exchange rate is ambiguous. I(2) property of the price differential might be due to the structural
change in US prices. This was also confirmed by the findings of I(1) analysis, which shows the need for a linear trend in a cointegrated vector understood as an inflation differential between Germany and U.S.A. The linear trend might now indicate the increased anti-inflation credibility of the FED which has decreased the inflation differential between countries.

The empirical analysis of the first I(1) data set was based on the PPP transformed vector, where the PPP is an I(1) variable. The joint restrictions were not data consistent in I(2) data set and some information is lost by moving to I(1). Thus, we have to admit that PPP transformation is a little problematic when implied restrictions are not statistically acceptable. Until we have full-fledged I(2) program there is not much we can do. However, I(2) analysis given in this study makes clear the importance of complete I(2) analysis in order to understand fully the relationship between the nominal exchange rate and prices.

Using the data vector with PPP transformation we did not find any evidence for Harrod-Balassa-Samuelson hypothesis. Based on this result and the well known problems of the second component of Harrod-Balassa-Samuelson hypothesis (PPP for tradables), we extended our data vector with the oil price variable. Finally, we tested whether some of the basic economic hypotheses can be given a precise statistical formulation. Test results clearly shows that we can accept the Harrod-Balassa-Samuelson hypothesis if oil price is included in the data vector. This is consistent with the hypothesis of no-arbitrage condition if there is uncertainty as to the permanence of the shock causing relative tradable price changes. Similarly, we are not able find stationary cointegration vector, if only oil price is included in the vector. Thus, we can conclude that both sources of non-stationarity have to include in the estimation in order to get complete understanding of deviations from purchasing power parity.

Most of the time cointegration analysis rejects the hypothetical coefficients. This seems to imply that other variables than those included in the analysis might play a role and only with a much biller system is it possible to obtain a model that satisfies completely both statistical and economic interpretability. For example, as shown in Juselius and MacDonald (2000), interest rates are also very important for PPP. This is intensified in our model with the linear trend in inflation differential equation. The long term interest rate would give us a more convenient way to model decreased inflation expectations. Thus, it would be interesting to extend our model with the asset market variables. This would give us also a more precise estimate of the equilibrium real exchange rate level. This is especially important because the discussion of economic rationality of weak euro has
focused mainly on interest rate differential. The asset market approach together with the productivity differential and oil price discussed in this study would give us more complete understanding of the equilibrium level to which the economic adjustment forces pull the exchange rate between the German mark and the U.S. dollar.
REFERENCES:


International Statistical Yearbook (1995), IMF.


Juselius, K. and Gennari, E. (1998), Dynamic modeling and structural shift: Monetary transmission mechanisms in Italy before and after EMS. Unpublished manuscript, European University Institute.

Juselius, K. and Toro, (1999), The effects of joining the EMS; Monetary transmission mechanism in Spain. Discussion Paper 22, University of Copenhagen.


Krugman, P. (1985), Is strong dollar sustainable? In; The U.S. dollar - Recent developments, outlook and policy options ( Federal Reserve Bank, Kansas City) 103-133.


Krugman, P. (1989), Exchange rate instability, MIT Press; MA.


APPENDIX 1

The data set used consists of quarterly time series observations from 1975:2 to 1998:1 for Germany and the United States. The source of the data has mainly been the IMF International Financial Statistic (IFS). The productivity data is provided by the Bureau of Labor Statistics. Following variables are considered:

**Nominal exchange rate** is average-for-quarter observations defined as the German mark price of one US dollar.

**Prices** are consumer price indices for Germany and the U.S.A.

The average product of labor in manufacturing is used to measure productivity. Unfortunately, the equivalent measure for the non-tradables sector is not available. Thus, the measure productivity may be appropriate only under the further assumption that trend movements in relative productivity in services are insignificant in the U.S.A. and Germany.

The choice of the average product of labor differs from most of the recent literature, which uses total factor productivity as a proxy for productivity. This choice is not innocuous since labor shedding may introduce substantial differences between changes in average labor productivity and changes in total factor productivity. However, as Ganzoneri, Cumby and Diba (1999), have pointed out, the common habit of describing movements in total factor productivity using Solow residuals is problematic because interpreting movements in Solow residuals as exogenous supply shocks is far from perfect. Another important surplus of average labor productivity compared to total factor productivity is that it holds for a broader class of technologies than the Cobb–Douglas production function, which is used to compute Solow residuals.

To be precise, our measure of productivity is an index of labor productivity, constructed from real output per man hour in manufacturing in the U.S.A and Germany. As discussed above, a drawback to this measure of productivity is that it does not control for differences in investment rates, but on

---

50 Note that the construction of productivity series are not completely similar. Output series for Germany is based on value-added basis but the quarterly U.S. manufacturing output series is based on a sectoral output basis rather than a value-added basis. However, this should not prevent us to find empirically relevant relations between variables.

the other hand it avoids the many difficulties involved on measuring capital.\textsuperscript{52} The comparisons of productivity data are limited to trend measures only; no reliable comparisons of levels of manufacturing productivity are not available.\textsuperscript{53} Under the assumption that labor productivity in manufacturing reflects overall productivity in traded goods, we can assume that labor productivity estimates provide a less problematic measure of existing productivity differentials in tradables in the U.S.A and Germany than total factor productivity estimates.

**Oil price** is the quarterly average of the spot price of oil in US dollars, deflated by the US CPI. Although we use real oil price we cannot exclude the possibility that the construction of price indexes can affect on real exchange rate movements if oil price is weighted differently in the United States than in Germany. Suppose that

\begin{align*}
  p_t^T &= \theta \text{oil}_t + (1-\theta)p_t^2 \quad \text{and} \\
  p_t^{*2} &= \pi \text{oil}_t^* + (1-\pi)p_t^{2*}
\end{align*}

where $\theta$ and $\pi$ are weights in home and foreign country (foreign marked with *) price indicies, $p_t^T$ is a price index of tradable goods, $\text{oil}_t$ is an oil price index and $p_t^{2}$ non-oil price index. Lower case letters denote variables in logarithms. Even if the law of one price for each good held, if $\theta \neq \pi$, then $p_t^T - p_t^{*2}$ will change as $\text{oil}_t$ moves relative to $p_t^{2}$. However, Engel (1999) recalculated several traded goods price indices (including Germany) using the U.S. weights and found only little real exchange rate effect from using different weights. Thus, oil price seems to affect real exchange rates by some other way than the different index weights alone.

\textsuperscript{3} Using average labor productivity we do not need data on sectoral capital stocks, which are likely to be quite unreliable.

\textsuperscript{53} To compare manufacturing output across countries, a common unit of measurement would be needed, such as the U.S. dollar. Market exchange rates are not suitable as a basis for comparing output levels. What is needed is reliable PPP. Reasonably reliable PPP is available only for total gross domestic product not for manufacturing product.
FIGURE 1. NOMINAL EXCHANGE RATE

FIGURE 2. DIFFERENCED NOMINAL EXCHANGE RATE
FIGURE 3. REAL EXCHANGE RATE

FIGURE 4. REAL OIL PRICE
FIGURE 5. GERMAN PRICE INDEX

FIGURE 6. GERMAN INFLATION
FIGURE 7. U.S. PRICE INDEX

FIGURE 8. US. INFLATION
APPENDIX 2

COINTEGRATION VECTOR 1

COINTEGRATION VECTOR 2
COINTEGRATION VECTOR 3

\[ \beta_3^* = Z_k(t) \]

\[ \beta_3^* = R_k(t) \]