HOUSING POLICY AND REDISTRIBUTION

Sanna Tenhunen
Matti Tuomala

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Housing Policy and Redistribution

Sanna Tenhunen
University of Tampere and FDPE

and

Matti Tuomala
University of Tampere

Abstract: The potential redistributive role of the housing subsidies has got a relatively little attention in the literature. However, non-linear housing subsidy schemes are commonly used by many countries. We consider the question of the optimal tax treatment of housing in a model where agents differ in two dimensions: ability and the housing price they face. The problem is also solved numerically. In addition to a general four-type model, we also considered two extensions: endogenously determined housing prices and paternalistic government. The key lesson from the paper is that when individuals differ also in access to housing, the housing subsidy schemes have a redistributive role and they are non-linear. A justification for subsidizing or taxing housing holds also without introducing any merit good argument to the analysis; in the optimal tax model where agents differ in their abilities and access to housing it is optimal to tax and subsidize housing, even with separable preferences.

Keywords: Housing subsidies, two-dimensional heterogeneity, optimal taxation

JEL: H23, H24, D62
1 Introduction

Housing is the most important item in family budgets and housing outlays are in a number of ways influenced by the government’s policy either in the form of tax relief or direct subsidies. It is argued that the higher the housing outlays, the lower the taxable capacity, and therefore there should be a tax deduction or a tax credit for housing outlays. On the other hand there is an argument that tax deductions are unacceptable because under progressive income taxation it means that different individuals pay different effective prices for housing and the price is lowered more for the rich than for the poor. Therefore it has been proposed that a tax credit would be preferable to tax deductions. A tax credit is simply equivalent to a proportional subsidy for housing: it involves an equal reduction in the price of housing to all individuals. However, in many countries housing costs are taken into account in basic maintenance supports. Thus, the subsidy is not necessarily equal to all, but in effect housing subsidies might be non-linear.

The interrelation of a general subsidy with other instruments of public policy may be analyzed within the framework of optimal taxation. If there is a freely variable non-linear income tax, people differ only with respect to their earning potential and the weak separability between labour and other commodities holds, then neither tax deductibility nor a tax credit would be desirable (see Atkinson, 1977 or Atkinson-Stiglitz, 1980)). Thus, the desirability of tax deductions or tax credit must be based on the recognition of the facts that there are long-run inequalities in access to housing, the weak separability does not apply in preferences and there are restrictions in the use of non-linear income tax schedules. The third of these is explored by Atkinson (1977) who considers a model where the individuals have identical utility functions but they differ in their earning abilities (wage per hour). He shows in the case of linear taxation, when all goods are normal and substitutes in the Hicksian sense, the optimal rate of subsidy on housing is strictly less than the rate of income taxation. He also considers the case where the price of housing varies across individuals\(^1\), but so that housing prices and

\(^1\)King (1980) provides an empirical support in the UK that the price of housing services
wages are independent. In housing markets with market imperfections this is not necessarily true. It is possible, that more able individuals have higher earning prospects and might thus have better access to mortgages required to owner-occupied housing with lower unit costs. There is some evidence from Finland (see Lyytikäinen-Lönnqvist, 2005), that the correlation between income and housing price is negative. Thus, at an intuitive level, there might be some support for a negative correlation, but the direction of the dependence between household income and housing price is not obvious.

We examine the role of housing subsidy schemes as a redistributive mechanism when tax policies are not artificially restricted to be linear. They are optimized given the structure of information in the economy. The underlying information structure is the standard one in the optimal taxation literature, following the footsteps of Mirrlees (1971, 1976). Also Cremer and Gahvari (1998) study the question of optimal taxation of housing in this set up. In their model there are two types of housing goods (low and high quality) for which the agents have different tastes. They assume that tastes for housing and wages are positively correlated, and consider a two-type model with one particular configuration of binding self-selection constraint. We in turn employ a two-dimensional case where individuals differ both by their productivity and the unit price of housing they face. We assume a discrete distribution of types, which leads to a four-type model. This multidimensional heterogeneity in agents’ characteristics is a realistic assumption but it complicates the analysis notably, as is discussed e.g. in Boadway et al. (2002). The biggest challenge in a multidimensional screening problem is the choice of the binding self-selection constraints. There are some analytical studies in a discrete case with two-dimensional heterogeneity, but they are usually simplified further to a three-type case with strong assumptions of the bindingness of self-selection constraints. Instead of choosing the binding self-selection constraints a priori we include all of them to the optimization

\[^2\]A discrete three-type models are considered e.g. in Cuff (2000) and Blomquist and Christiansen (2004). A continuous case is considered numerically in Tuomala and Tarkiainen (1999; 2007).
problem. To gain better understanding of the housing subsidies and binding constraints we also solve the problem numerically. A two-type model with perfect correlation between productivity and housing prices is considered as a special case.

In the previous analysis, assumptions were chosen so that they rule out changes in pre-tax (subsidy) housing cost. There is no good reason to think that distribution of housing prices is fixed and given. The endogenous price determination is shown to have essential effects on the optimal tax policy (Naito, 1999; Micheletto, 2004). In this paper we extend the analysis of the optimal tax treatment of housing to take into account the possibility that taxes (subsidies) affect housing prices.

Housing is typically analyzed in the literature so that there is nothing to distinguish housing services from any other commodity. Atkinson (1977) gives two interesting examples of this kind of views from Friedman and Engels3. But the politics of government housing policy suggest that the commodity is, indeed, a “special” or merit commodity. Rosen (1985) discusses both the efficiency and equity arguments to subsidize housing. The question of the optimal tax treatment for housing is closely related to analysis of merit goods, starting from Musgrave (1959). The optimal tax treatment of merit goods has been analyzed e.g. by Besley (1988), Racionero (2001), Schroyen (2005). Also ‘specific egalitarianism’, originally introduced by Tobin (1970) suggests that intervening to the distribution of certain goods essential to life, like housing, might be beneficial. Thus paternalism may be more plausible characterization of housing policy in many countries. It may take a form that government wants all households to receive a certain minimum level of housing services. As an example we consider a specific type of paternalism, where housing is introduced directly to the social welfare function. The difference between government’s and individuals valuation of housing is determined with help of marginal rates of substitution, following the lines of

3“Public housing is proposed not on the ground of neighbourhood effects but as a means of helping low-income people. If this is the case, why subsidize housing in particular” (Friedman). “The rent agreement is quite an ordinary commodity transaction which is... of no greater and no lesser interest to the worker than any other commodity transaction, with the exception of that which concerns the buying and selling of labour power” (Engels).

The structure of the paper is as follows. Section 2 we introduce the model. Section 3 considers a four-type version of the Mirrlees model with welfarist government and computes also some numerical examples. No a priori assumptions of the binding self-selection constraints are made in our numerical analysis. We simply determine them by solving this problem numerically. In Section 4 it is shown that endogenous housing prices affects the optimal housing subsidy schemes. The problem is considered both in general four-type model and as a simplified, more tractable special case with two types. In section 5 we briefly consider the case of paternalistic government. Finally, Section 6 concludes.

2 The model

In the long run the access to housing is likely to be unequal. This gives good grounds to consider a case where individuals differ not only with respect to their earning potential but also with respect to the price of housing they face. The model we employ is a two-dimensional extension to a discrete interpretation of the Mirrlees model introduced in Stiglitz (1982) and Stern (1982). We assume that the economy consists of different types of individuals who are distinguished both by earnings abilities, $n$ and by housing costs per unit, $s$. Both wages and housing costs are assumed to be determined exogenously, i.e. we assume perfectly elastic supply of labour and housing.\footnote{This assumption will be relaxed in Section 4, where we consider a case with endogenous housing prices.} Individuals are assumed to have separable preferences in goods, housing and labour supply given by

\[ U = u\left(c^i\right) + v\left(\frac{H^i}{s^i}\right) - \psi\left(\frac{z^i}{n^i}\right) \tag{1} \]

where $c$ is other consumption, $H = sh$ is housing expenditure with $h$ representing the quantity of housing, and $z (= ny)$ denotes gross income with $y$ giving the labour supplied. $c$ is taken as the numeraire and the pre-tax price...
of the housing is unity. In the absence of housing price variations these preferences imply that housing subsidies (or taxes) have no redistributive role. The separability assumption makes it possible to isolate the significance of variations in access to housing.

The individual budget constraint can be written as follows

\[ c^i + K (s^ih^i) = n^iy^i - T (n^iy^i) \]  \hspace{1cm} (2)

where \( K \) denotes the expenditure on housing. This constraint allows for non-linearity of both the income tax, \( T \), and housing subsidy (or tax) schedule, \( K \). Clearly it is unrealistic to assume that a non-linear tax schedule is administratively feasible for all commodities as preventing the resale is impossible. However, in many countries the governments have information on personal housing consumption, especially for owner-occupiers. This makes it possible that housing can be subsidized for some households at a rate which varies according to how much they spend it.

The condition for the individual maximization of utility subject to (2) implies that (subscripts are partial derivatives with respect to appropriate arguments)

\[ v_h - sK'u_c = 0 \quad , \quad \psi_z + u_c (1 - T') n = 0 \]  \hspace{1cm} (3)

From these we can solve marginal income tax \( T' = 1 - \frac{\psi_z}{u_c} \) and marginal housing subsidy \( K' = \frac{v_h}{s'u_c} \).

\section{A general four-type case with unequal access to housing}

The correlation between productivity and housing prices is not self-evidently perfect. With an imperfect dependence between these two variables we end up having four different types. They are denoted by numbers 1-4 according to Table 1.
We assume that there are \( N^i \) number of each type, such that \( \sum N^i = 1 \). Governments’ problem is to design a tax system with possibly a non-linear income tax \( T \) and a housing subsidy scheme \( K \). There is asymmetric information in a sense that tax authority is informed neither about individual skill levels, labour supply nor the housing price they face. It can only observe before-tax income, \( z \) and housing expenditure \( H \).

With an utilitarian government the optimal income and housing subsidy schedules are chosen according to a problem

\[
\max \sum N^i \left[ u(c^i) + v \left( \frac{H^i}{s^i} \right) - \psi \left( \frac{z^i}{n^i} \right) \right]
\]

subject to the self-selection constraints given by

\[
u(c^j) + v \left( \frac{H^j}{s^j} \right) - \psi \left( \frac{z^j}{n^j} \right) \geq \hat{u}(c^j) + \hat{v} \left( \frac{H^j}{s^j} \right) - \hat{\psi} \left( \frac{z^j}{n^j} \right)
\]

for \( i, j = 1, 2, 3, 4 \) and \( i \neq j \) (5)

where the terms with a 'hat' refer to mimickers, and the government revenue constraint

\[
\sum N^i (z^i - c^i - H^i) = R
\]

where \( R \) is some exogenous revenue requirement. The first order conditions with respect to \( c^i, H^i \) and \( z^i \) can be given in a general form as

\[
\text{Notice that the utility functions of a mimicker and true type person are indentical, the 'hat' is marked just for tractability.}
\]
\[ N^i u_c - \lambda N^i + \sum_j \left( \mu^{ij} u_c - \mu^{ji} u_c \right) = 0 \]  \hspace{1cm} (7) 

\[ N^i v_H \frac{1}{s^i} - \lambda N^i + \sum_j \left( \mu^{ij} v_H \frac{1}{s^i} - \mu^{ji} v_H \frac{1}{s^j} \right) = 0 \]  \hspace{1cm} (8) 

\[ -N^i \psi_z \frac{1}{n^i} + \lambda N^i - \sum_j \left( \mu^{ij} \psi_z \frac{1}{n^i} - \mu^{ji} \psi_z \frac{1}{n^j} \right) = 0 \]  \hspace{1cm} (9) 

where \( \lambda \) and \( \mu^{ij} \) are the Lagrange multipliers for the budget constraint and the self-selection constraints binding type \( i \) from mimicking type \( j \), respectively.

By manipulating the first order conditions (7) and (9) gives the standard results for the marginal income tax rates:

\[ T^{iv} = 1 - n^i \frac{N^i + \sum_j \mu^{ij} - \sum_j \mu^{ji}}{N^i + \sum_j \mu^{ij} - \sum_j \frac{\mu^{ji} n^i}{n^j}} \]  \hspace{1cm} (10) 

Equivalently, combining conditions (7) and (8) gives the marginal subsidy for housing price:

\[ K^{iv} = \frac{N^i + \sum_j \mu^{ij} - \sum_j \mu^{ji}}{N^i + \sum_j \mu^{ij} - \sum_j \frac{\mu^{ji} s^i}{s^j}} \]  \hspace{1cm} (11) 

The marginal housing subsidy is zero for type \( i \) only when there is no difference in housing prices, i.e. \( s^i = s^j \) for all \( j \). The magnitudes of the taxes and subsidies depend on the binding self-selection constraints. Without a priori assumptions it cannot be determined, which of the possible 12 self-selection constraints given in (5) are the ones that are binding in the optimum. To illustrate a possible outcome, we consider next a numerical solution for the problem.

**Numerical solution**

The utility function considered is CES-form: \( U = -\frac{1}{c} - \frac{1}{h} - \frac{1}{1-v} \). We first assume that there is a uniform distribution of types in the economy, i.e.
the fraction of each type i is $N^i = 0.25$. The wage rates are 5 for the low productivity types and 9 for the high productivity types and the unit prices of housing are 0.02 and 0.04. The prices are chosen so that the problem is numerically solvable and the resulting fractions of housing costs remain on a plausible level, approximately one fifth or less of the net income. The results of numerical solution are given in Table 2.

Our numerical results bring out several interesting points of the optimum. The first thing to notice is that the pattern of the binding self-selection constraints is not simple. For example, the self-selection constraints are not binding for either of the low-skilled types. The fact that self-selection constraints are not binding at the same time to both directions (meaning $(i, j)$ and $(j, i)$ are not simultaneously binding) implies that the optimum is separating optimum, no pooling seems to occur.

Numerical solution gives us also the marginal tax rates. There is a positive marginal income tax for both low-skilled types and zero marginal tax rates for both high-skilled types. More interesting results are the marginal housing subsidies given by $K'$, where $K' = 1$ implies that there is no distortion in the housing. There is no distortion for types 2 and 3, whereas there is a subsidy for the low-skilled type with a high unit price of housing (type 1) and a tax for the high-skilled type with a low unit price of housing (type 4).

<table>
<thead>
<tr>
<th></th>
<th>$U$</th>
<th>$c$</th>
<th>$h$</th>
<th>$y$</th>
<th>$T'$</th>
<th>$K'$</th>
<th>$H/c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>type 1</td>
<td>-2.2327</td>
<td>1.3846</td>
<td>7.2576</td>
<td>0.2715</td>
<td>0.2241</td>
<td>1.2468</td>
<td>0.2097</td>
</tr>
<tr>
<td>type 2</td>
<td>-2.1561</td>
<td>1.4331</td>
<td>10.1333</td>
<td>0.2645</td>
<td>0.1936</td>
<td>1</td>
<td>0.1414</td>
</tr>
<tr>
<td>type 3</td>
<td>-2.0376</td>
<td>2.0612</td>
<td>10.3061</td>
<td>0.3129</td>
<td>0</td>
<td>1</td>
<td>0.2000</td>
</tr>
<tr>
<td>type 4</td>
<td>-1.9687</td>
<td>2.1054</td>
<td>14.5153</td>
<td>0.2982</td>
<td>0</td>
<td>0.9759</td>
<td>0.1379</td>
</tr>
</tbody>
</table>

Table 2: Numerical solution

We assumed above a uniform distribution of types implying that the correlation between ability and housing costs are zero. However, it is possible, through

\footnote{For example in Finland housing costs vary between 15 and 20 per cents of net income (Lyytikäinen and Lönnqvist, 2005).}
that more able individuals have higher earning prospects and might thus have better access to mortgages required to owner-occupied housing with lower unit costs. There is also some evidence from Finland (see Lyytikäinen-Lönnqvist, 2005), that the correlation between income and housing price is negative. Thus, at an intuitive level, there might be some support for a negative correlation, although the sign of the correlation between household income and housing price is not obvious. Therefore, Tables (3) and (4) present numerical solution to the problem with assumptions of (imperfect) negative and positive correlation induced by a different distribution of types in the economy.7 The results are otherwise consistent with uniform distribution above, except for the housing subsidy rates. With negative correlation the subsidy for type 1 seems to disappear, whereas there is now tax for both types with low housing price. With positive correlation between ability and housing costs there is a subsidy for type 1, whereas the rest of the types are undistorted.

<table>
<thead>
<tr>
<th>Type</th>
<th>(N^t)</th>
<th>(U)</th>
<th>(c)</th>
<th>(h)</th>
<th>(y)</th>
<th>(T')</th>
<th>(K')</th>
<th>(H/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>type 1</td>
<td>0.1</td>
<td>-2.2347</td>
<td>1.3634</td>
<td>6.8171</td>
<td>0.2617</td>
<td>0.2614</td>
<td>1</td>
<td>0.2000</td>
</tr>
<tr>
<td>type 2</td>
<td>0.4</td>
<td>-2.1369</td>
<td>1.4608</td>
<td>10.2263</td>
<td>0.2617</td>
<td>0.1738</td>
<td>0.9902</td>
<td>0.1400</td>
</tr>
<tr>
<td>type 3</td>
<td>0.4</td>
<td>-2.0503</td>
<td>2.0485</td>
<td>10.2425</td>
<td>0.3172</td>
<td>0</td>
<td>1</td>
<td>0.2000</td>
</tr>
<tr>
<td>type 4</td>
<td>0.1</td>
<td>-1.9525</td>
<td>2.1567</td>
<td>10.2263</td>
<td>0.2811</td>
<td>0</td>
<td>0.7842</td>
<td>0.0948</td>
</tr>
</tbody>
</table>

Bound self-selection constraints: (1,2), (3,1), (3,2), (3,4) and (4,2)

Table 3: Numerical solution when correlation between ability and housing price is negative

<table>
<thead>
<tr>
<th>Type</th>
<th>(N^t)</th>
<th>(U)</th>
<th>(c)</th>
<th>(h)</th>
<th>(y)</th>
<th>(T')</th>
<th>(K')</th>
<th>(H/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>type 1</td>
<td>0.4</td>
<td>-2.2348</td>
<td>1.3742</td>
<td>7.7148</td>
<td>0.2740</td>
<td>0.2288</td>
<td>2.0898</td>
<td>0.2246</td>
</tr>
<tr>
<td>type 2</td>
<td>0.1</td>
<td>-2.1612</td>
<td>1.4293</td>
<td>10.1064</td>
<td>0.2661</td>
<td>0.1940</td>
<td>1</td>
<td>0.1414</td>
</tr>
<tr>
<td>type 3</td>
<td>0.1</td>
<td>-2.0369</td>
<td>2.0620</td>
<td>10.3098</td>
<td>0.3127</td>
<td>0</td>
<td>1</td>
<td>0.2000</td>
</tr>
<tr>
<td>type 4</td>
<td>0.4</td>
<td>-1.9721</td>
<td>2.1000</td>
<td>14.8494</td>
<td>0.3000</td>
<td>0</td>
<td>1</td>
<td>0.1414</td>
</tr>
</tbody>
</table>

Bound self-selection constraints: (3,1), (3,4), (4,1) and (4,2)

Table 4: Numerical solution when correlation between ability and housing price is positive

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7Correlation coefficient used in these calculations is -0.6 and 0.6, respectively.
One interesting outcome from the results is that the optimal marginal housing subsidy seems to be sensitive to the assumptions of the structure of the economy. Next we consider in more detail how correlation between ability and housing price affects housing subsidies (figure 1). It can be concluded that when the correlation between ability and housing costs is sufficiently negative, types 2 and 4 (types with low housing price) are marginally taxed while types 1 and 3 (types with high housing price) are nondistorted. As the correlation becomes less negative, type 2’s marginal tax vanishes and type 1 starts to get marginal subsidy. Finally, as correlation becomes positive and sufficiently large, also the marginal tax on type 4 disappears. The critical values at which the the marginal subsidy or tax starts/disappears are presented in figure. It is also worth noticing that the marginal subsidy on housing for type 1 might be substantial, when correlation between ability and housing costs becomes sufficiently large and positive.

A special case

Some clarifying analytical results can be derived in a simplified framework where the number of types is reduced to two. This characterization of the model is achieved by assuming dependence between ability and housing price to be perfect. We consider next a two-type version of the model with a perfect negative correlation between ability and housing prices. The two types of individuals are indexed according to Table 1 but assuming that there are no type 2 or type 3 individuals at all, i.e. we have type 1 = \((n_L, s_H)\), and type 4 = \((n_H, s_L)\). We also make the traditional assumption that only the self-selection constraint for high-ability type is binding. The problem can now be written as

\[
\max \sum_{i=1,4} N^i \left( u(c^i) + v \left( \frac{H^i}{s^i} \right) - \psi \left( \frac{z^i}{n^i} \right) \right)
\]  

\text{(12)}

\footnote{A closely related analysis is carried through in Cremer and Gahvari (1998), who assume that the taste for low and high quality housing is positively correlated.}

\footnote{This assumption can be confirmed numerically.}
Figure 1: Dependence of the marginal subsidy on housing on the correlation between ability and housing costs.

such that

\[ \sum_{i=1,4} N^i (z^i - c^i - H^i) = R \]  

and

\[ u(c^4) + v \left( \frac{H^4}{sL} \right) - \psi \left( \frac{z^4}{nH} \right) \geq \tilde{u}(c^1) + \tilde{v} \left( \frac{H^1}{sL} \right) - \tilde{\psi} \left( \frac{z^1}{nH} \right) \]  

Manipulation of the first order conditions\(^{10}\) gives the marginal tax rates. The marginal income tax rates are \( T^1 < 0 \) and \( T^4 = 0 \) implying that the labour supply decision of the high-skilled types is undistorted, while there

\(^{10}\)The first order conditions are given in Appendix B.
is a positive marginal tax on type 1’s labour income. The marginal housing subsidies imply that $K^{4} = 1$, i.e. the housing price is neither taxed nor subsidized for type 4, whereas for type 1 there is a distortion given by

$$K^{1} = 1 + \frac{\mu H}{\lambda N} \left( \frac{1}{s^{L}} - \frac{1}{s^{H}} \right)$$

(15)

In the empirically plausible case with $s^{L} < s^{H}$ the term in the right hand side of equation (15) is positive, so for a low skilled individual we obtain an unambiguous marginal subsidy. This result is in accordance with the numerical results found in a general four-type case. Following the general result, there is no housing subsidy either for type 1 if and only if $s^{L} = s^{H}$.

Because of the two-dimensional heterogeneity, a subsidy for type 1 is an effective way to relaxing an otherwise binding self-selection constraint. This is because even under separability mimicker and mimicking individual do not consume the same amount of housing. In other words, when people differ both in skills and housing prices there is a case for distortionary taxation of housing even with separable preferences. We have also “no distortion at the top”-result here. This is simply because in equilibrium no one wants to mimic the high-skilled individual with a low unit price of housing (type 4). Thus no gain can be achieved by taxing or subsidizing his marginal consumption.

One important implication of this model is that there is two separate schedules; a non-linear income tax schedule depending only on labour income, $z$ and a non-linear housing subsidy scheme depending only on housing expenditure, $H$. Hence the optimum policy can be implemented through two separate functions.

4 **Endogenous housing prices**

The assumption of perfectly elastic supply of housing might be acceptable at the national level, but not at the local level. When the elasticity of supply of housing is not infinite, subsidies that alter the demand for housing will at the same time alter the equilibrium housing prices. Therefore, housing prices are endogenous to the government optimization problem in a more
realistic specification we will consider next by giving up the assumption of the exogenously determined costs of housing services, \( s \).

We continue to assume, without a loss of generality, that even if housing prices are endogenously determined the order still remains so that \( s^H > s^L \). Now we assume that the two groups demand different kinds of housing services \((h^L, h^H)\). The relative price will then depend on the relative demands of the two type of housing services. The ratio of housing prices, is now a function of housing expenditures \( H \) and housing consumption \( h \), defined as \( \Omega^{ij} = \frac{z^i}{z^j} = \frac{H^i}{H^j} \frac{h^j}{h^i} \). Government’s problem is to maximise the sum of utilities (4) subject to the government budget constraint (6) and the self-selection constraints that are now given by

\[
N^i u(c^i) + v \left( \frac{H^i}{s^i} \right) - \psi \left( \frac{z^i}{n^i} \right) \\
- \hat{u} \left( c^j \right) - \hat{v} \left( \Omega^{ji} \left( H^i, H^j \right) \frac{H^j}{s^j} \right) + \hat{\psi} \left( \frac{z^j}{n^j} \right) \\
\text{for } i, j = 1, 2, 3, 4 \text{ ja } i \neq j \quad (16)
\]

The first order conditions with respect to \( c, H \) and \( z \) are

\[
N^i u_c - \lambda N^i + \sum \left( \mu^{ij} u_c - \mu^{ji} \hat{u}_c \right) = 0 \quad (17)
\]

\[
N^i v_H \frac{1}{s^i} - \lambda N^i + \sum \left[ \mu^{ij} \left( v_H \frac{1}{s^i} - \hat{v}_H \frac{H^j}{s^j} \frac{d\Omega^{ji}}{dH^i} \right) \right. \\
\left. - \mu^{ji} \hat{v}_H \left( \frac{\Omega^{ij}}{s^i} + \frac{H^i}{s^i} \frac{d\Omega^{ij}}{dH^i} \right) \right] = 0 \quad (18)
\]

\[
-N^i \psi_z + \lambda N^i - \sum \left( \mu^{ij} \psi_z \frac{1}{n^i} - \mu^{ji} \hat{\psi}_z \frac{1}{n^j} \right) = 0 \quad (19)
\]

\(^{11}\)Also the wage rates might have been considered as endogenous here. However, as we use a separable utility function, the endogenousness of wages would have affected only the marginal income tax rates leaving the marginal housing subsidies unchanged.
The marginal tax rates are not affected by the relaxation of the assumption of housing price formation. Thus they are still given by (10). Instead, the marginal subsidies for housing are now different:

$$K^{ij} = \frac{N^i + \sum_j \mu^{ij} - \sum_j \mu^{ji}}{N^i + \sum_j \mu^{ij} \left(1 - H^i \Omega^{ij} \frac{d\Omega^{ij}}{dH^i}\right) - \sum_j \mu^{ji} s^i \left(1 + H^i \Omega^{ij} \frac{d\Omega^{ij}}{dH^i}\right)}$$ (20)

Compared to previous case there are now two terms in the denominator of (20), denoted by A and B, that differ from marginal subsidies for housing in the exogenous case (11). Because of the definition of $\Omega^{ij}$, we have $\frac{d\Omega^{ij}}{dH^i} = -\frac{\Omega^{ij}}{H^i} < 0$ and $\frac{d\Omega^{ij}}{dH^i} = \frac{\Omega^{ij}}{H^i} > 0$. Thus, it can be concluded, that compared to the exogenous case, the marginal housing subsidy for type $i$ is increased by the effect of the self-selection constraints preventing type $i$ from mimicking the other types (constraint attached with Lagrange multipliers $\mu^{ij}$) and it is decreased by the effect of the self-selection constraints preventig the other types from mimicking type $i$ (constraint attached with Lagrange multipliers $\mu^{ji}$).

A special case

Equivalently to earlier case, also here a more straight forward results can be obtained by considering a simplified model with perfect correlation between ability and housing prices. The two-type case follows earlier path: government maximizes (12) subject to the government budget constraint (13) and the self-selection constraint binding the high ability type given now by

$$u(c^4) + v(c^4) - \psi\left(\frac{H^4}{s^4}\right) - u(c^1) - \tilde{v}(\Omega^1 s^1) + \psi\left(\frac{z^1}{\mu^1}\right)$$ (21)
where $\Omega = \frac{s^H}{s^L} = \frac{H^1}{H^4}$. From the first order conditions\footnote{The first order conditions are given in Appendix B} we can solve the marginal subsidies for housing. We have now for type 1

$$K^{1'} = 1 + \frac{\mu \psi H}{\lambda N^1} \left( \frac{1}{s^L} - \frac{1}{s^H} \right) + \frac{H^1}{s^H} \frac{d \Omega}{d H^1}$$

(22)

and for type 4

$$K^{4'} = \frac{N^4 + \mu}{N^4 + \mu - \mu \left( \frac{\Omega}{H^4} \frac{d \Omega}{d H^4} \right)}$$

(23)

For type 1 there is a similar term inside the brackets in (22) than in earlier case. It is positive as a result of the assumption that $s^H > s^L$. The other term, denoted with a C is also positive, because $\frac{\partial \Omega}{\partial H^1} = \frac{\Omega}{H^1} > 0$. Thus, endogenizing the housing prices increases the marginal subsidy of housing for type 1. For type 4, the high-skilled type, there is now an additional term in (23) compared to exogenous case, where housing decision were left undistorted. The term, denoted with a D is negative, as $\frac{\partial \Omega}{\partial H^4} = -\frac{\Omega}{H^4}$. Thus, $K^{4'}$ is smaller than one indicating that there is a housing tax for type 4. It can be concluded that when taking into account the general equilibrium effects on price determination government should be induced to increase (lower) the marginal tax (subsidy) rate on the high skilled person. By doing so the government decreases the housing consumption of the high-skilled individuals, which reduces the before-tax (subsidy) housing price differentials and thus relaxes the self-selection constraint.

5 Paternalism and housing

Housing differs from the other commodities in several aspects. First, it provides externalities, like the quality of the neighbourhood, as discussed e.g. in Rosen (1985). Second, housing can be thought to be a necessity, that
from equity point of view should be available to all citizens. Thus, housing can be interpreted as a merit good in a sense introduced by Musgrave (1959). Also 'specific egalitarianism', originally introduced by Tobin (1970) suggests that intervening to the distribution of certain goods essential to life, like housing, might be beneficial. In fact in many countries housing is a strongly subsidized commodity suggesting that policy maker prefers to affect the individuals’ choices concerning housing. Therefore paternalism may be a plausible characterization of housing policy.

Paternalism may take several different forms. For example, government might think that all households should receive a certain minimum level of housing resources. Hence we might want to introduce housing directly into the social objective function. We apply this view of paternalism to our model by assuming that the government includes the gains from housing directly with some function $P$, whereas individuals continue to value housing with utility function $v$. We consider an example, where the utility of the government is given by

$$u(c) + P\left(\frac{H}{s}\right) - \psi\left(\frac{z}{n}\right)$$

while individuals continue to use utility given in (1).

The difference in the valuation of housing can be formulated in terms of marginal rates of substitution between housing and other consumption, given by $MRS^g = -\frac{u_c}{u}$ for the government and $MRS^p = -\frac{u_c}{u}$ for the individuals. Government’s higher valuation of the housing implies that $MRS^g > MRS^p$, which is exactly the same condition for merit good as used e.g. in Blomquist and Micheletto (2006). However, as we use a separable form of the utility function, the partial derivative $u_c$ is equal in both government’s and individual’s marginal rate of substitution. Thus the condition for the government’s valuation of housing to exceed that of the individual’s can be reduced to form $P_H < v_H$.

Government’s problem is to maximize

The optimal tax treatment of merit goods has earlier been analyzed e.g. in Sandmo (1983), Besley (1988), Racionero (2001), Schroyen (2005) and Blomquist and Micheletto (2006).

13The optimal tax treatment of merit goods has earlier been analyzed e.g. in Sandmo (1983), Besley (1988), Racionero (2001), Schroyen (2005) and Blomquist and Micheletto (2006).
\[
\sum N^i \left( u(c^i) + P \left( \frac{H^i}{s^i} \right) - \psi \left( \frac{z^i}{n^i} \right) \right)
\]  
\hspace{1cm} \text{(25)}

subject to the government budget constraint (6) and the self-selection constraints (5). The first order conditions with respect to \(c\) and \(z\) remain the same as earlier, and are given by (7) and (9), whereas the condition with respect to \(H\) is now

\[
N^i P_H \frac{1}{s^i} - \lambda N^i + \sum_j \left( \mu^{ij} v_H \frac{1}{s^j} - \mu^{ji} v_H \frac{1}{s^j} \right) = 0
\]  
\hspace{1cm} \text{(26)}

Because the assumption of paternalism in this example affects only the utility from housing and we have adopted separable utility function, the marginal income tax rates remain unchanged and are given by (10). The marginal subsidy for housing can be solved from first order conditions (7) and (26):

\[
K^{\nu} = \frac{N^i + \sum_j \mu^{ij} - \sum_j \mu^{ji} \frac{1}{\lambda s^j} \left( 1 - \frac{1}{\lambda s^i} (P_H - v_H) \right)}{N^i + \sum_j \mu^{ij} - \sum_j \mu^{ji} \frac{1}{s^j} \left( 1 - \frac{1}{\lambda s^i} (P_H - v_H) \right)}
\]  
\hspace{1cm} \text{(27)}

The marginal subsidy for housing again depends on the pattern of the binding self-selection constraints. In addition to that, there is now an additional term compared to general case given in (11), \(\frac{1}{\lambda s^i} (P_H - v_H)\). This term is negative as long as the marginal utility form an additional unit of housing from government’s view exceeds the marginal utility individuals perceive. Thus, it can be concluded that compared to the case with utilitarian government there is now a positive distortion for all types indicating a subsidy on housing. This is actually an intuitive result; the government can induce individuals to consume more housing only by subsidizing it (at the margin).

**A special case**

In the simplified two-type case with a perfect correlation between ability and housing costs we can derive analytically the marginal subsidies for housing. The government’s problem is now to maximize (12) subject to the government budget constraint (13) and the self-selection constraint binding the high
ability type given by (14). Manipulating the first order conditions to similar form as in the general two-type case we can write the conditions for the optimal marginal subsidies for housing for type 1 as

$$K^1 = 1 + \frac{\mu v_H}{\lambda N^3} \left( \frac{1}{s^L} - \frac{1}{s^H} \right) - \frac{1}{\lambda s^H} (P_H - v_H)$$

(28)

and for type 4 as

$$K^4 = 1 - \frac{1}{\lambda s^L} (P_H - v_H)$$

(29)

The additional terms in equations (28) and (29) for the marginal housing subsidy rate are analogous to a Pigouvian subsidy correcting an externality. When government valuates housing higher than individuals, we have $P_H < v_H$ and the last terms in (28) and (29) are positive indicating subsidization of housing. The marginal housing subsidy for type 1 is increased and type 4, that was earlier undistorted, is now subsidized as well.

6 Conclusions

The potential redistributive role of the housing subsidies has got a relatively little attention in the literature. However, non-linear housing subsidy schemes are commonly used by many countries. We consider the question of the optimal tax treatment of housing in a model where agents differ in two dimensions; ability and the housing price they face. The problem is also solved numerically. In addition to a general four-type model, we also considered two extensions: endogenously determined housing prices and paternalistic government.

Our results provide some support for a view that a general income tax based on earning abilities alone would not be sufficient for redistributive purposes when individuals differ also in access to housing. The pattern of the binding incentive compatibility constraints plays a crucial role in determining the optimal tax treatment of housing. The key lesson from the paper is that housing subsidy schemes have a redistributive role and they are non-linear.
Because of the two-dimensional heterogeneity, a distortion in housing price is an effective way to relax an otherwise binding self-selection constraint. This follows from the fact that even under separability mimicker and mimicking individual do not consume the same amount of housing.

One important implication of the model we considered is that there are two separate schedules; a non-linear income tax schedule depending only on labour income and a non-linear housing subsidy scheme depending only on housing expenditure. Hence the optimum policy can be implemented through separate functions and the administration of the two systems can be separated. This might eliminate the problem of a low rate of 'take up' or at least alleviate it. It also excludes means testing.

This paper has provided a rationale for distorting housing prices on both equity and efficiency grounds. A justification for subsidizing or taxing housing holds also without introducing any merit good argument to the analysis; in the optimal tax model where agents differ in their abilities and access to housing it is optimal to tax and subsidize housing, even with separable preferences. An important feature of our results is that they do not depend on the specific form of social welfare function, but they hold also with all constrained Pareto-efficient allocation.
References


Blomquist, S. and V. Christiansen (2004), 'Taxation and Heterogeneous Preferences', CESifo working papers.


Engels, F. (no date), 'The Housing Question', Martin Laurence.


Appendix A: Numerical simulations

**Procedure**

Numerical simulation is carried out with the Matlab-program. The function used (fmincon) solves the optimum of a multivariable function with constraints that may be linear or non-linear and equality or inequality constraints. Notice that as the optimization function allows also slack constraints, we are not restricted to a priori assumptions on the binding self-selection constraints. Thus, we have included all possible constraints to the optimization procedure and simply determined the binding constraints with help of numerical solution.

**Numerical tables**

In addition to those tables presented in text, we present here some useful numerical values given by numerical simulations. The numerical values for the Lagrange multipliers that are used in the analytical forms of the marginal tax rates on income and savings are reported here.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu^{12}$</th>
<th>$\mu^{13}$</th>
<th>$\mu^{14}$</th>
<th>$\mu^{21}$</th>
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<th>$\mu^{42}$</th>
<th>$\mu^{43}$</th>
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<td>0.0300</td>
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Table 5: Numerical values for Lagrange multipliers in general four-type case

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<th>$\mu^{13}$</th>
<th>$\mu^{14}$</th>
<th>$\mu^{21}$</th>
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<table>
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Table 6: Numerical values for Lagrange multipliers when correlation is negative
Next we consider how changing the assumptions of the structure of economy, wage rates and housing prices affects the results.

The distribution of types, reflecting also the correlation between the two heterogeneities, productivity and housing price are considered in the text. Changes in correlation seem to affect the results of marginal tax on housing (figure 1): with negative and sufficiently large correlation there seems to be a tax on the housing for types 2 and 4 with low housing price, and as the correlation becomes positive, these taxes vanish and only type 1 with high housing price is subsidized on the margin. Type 3 remains undistorted for all possible correlations.

The problems with numerical solution of a multidimensional screening problem were met when trying to alter the wage rates.\textsuperscript{14} It turns out that the problem is not solvable with low wage ratios. This results from several opposite self-selection constraints which make the matrix with constraints very close to being singular. This in turn causes a failure to optimization algorithm. At this point we acquiesce to the solvable cases: within the range of (1,10) the lowest wage ratio that is solvable is 0.5556 given by wage rates used in the text. For comparison Table 8 presents the numerical solution with higher wage ratio, given by wage rates of 5 and 6.

The result shows that when the wage ratio is higher, i.e. the wage difference is decreased, the optimality of subsidizing housing declines; the tax for type 4 disappears, but the subsidy for housing for type 1 seems to remain,\textsuperscript{14}

\textsuperscript{14}For a discussion of the problems with numerical solution of multidimensional screening problems, see e.g. Judd and Su (2006).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu^{12}$</th>
<th>$\mu^{13}$</th>
<th>$\mu^{14}$</th>
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<table>
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<td>0.1251</td>
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</table>

Table 7: Numerical values for Lagrange multipliers when correlation is positive

\textit{Sensitivity analysis}
although at lower level. Also the self-selection constraint binding high-skilled
type with higher housing price from mimicking the other high-skilled type
has become slack.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>c</th>
<th>h</th>
<th>y</th>
<th>T'</th>
<th>K'</th>
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<td>7.6957</td>
<td>0.3421</td>
<td>0.0285</td>
<td>1.0534</td>
<td>0.2146</td>
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<tr>
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<td>1.4716</td>
<td>9.9068</td>
<td>0.3307</td>
<td>0.0190</td>
<td>1</td>
<td>0.1346</td>
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<td>type 3</td>
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<td>8.4271</td>
<td>0.3321</td>
<td>0</td>
<td>1</td>
<td>0.2056</td>
</tr>
<tr>
<td>type 4</td>
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<td>1.6653</td>
<td>11.0788</td>
<td>0.3198</td>
<td>0</td>
<td>1</td>
<td>0.1331</td>
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</table>

Table 8: Numerical solution in a case with high wage ratio

<table>
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<th>μ^{13}</th>
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Table 9: Numerical values for Lagrange multipliers in a case with high wage ratio

Also the relative housing cost seems to affect optimal tax treatment of
housing. We varied the housing cost ratio between 0.2 and 1 keeping the
lower cost fixed at 0.02.\textsuperscript{15} Marginal subsidies on housing costs for types 2
and 3 remain at the level of 1 indicating no distortion for them. However,
marginal subsidy on housing on type 1 seems to alter substantially (figure 2).
It is interesting to notice, that the changes in the slopes of marignal subsidy
on housing for types 1 and 4 happen at the same points (see vertical lines
in figure 2). The subsidy on type 1 decreases as the difference in housing
prices decreases until it starts again rise at the same time (0.45) as type 4
begins to be taxed. The marginal tax on the housing of type 4 increases until
the housing price ratio reaches 0.63, and from there on it decreases back to

\textsuperscript{15}The results would be similar also when higher housing cost is kept fixed while the
lower cost varies.
unity implying no distortion. The second peak for type 1’s housing subsidy is again at the same point, with housing price ratio of 0.63, from there the subsidy decreases towards unity. Naturally, when there is no difference in housing price, i.e the ratio is one, none of the types is subsidized or taxed.

Figure 2: Dependence of the marginal subsidy on housing on the housing price ratio.
Appendix B: First order conditions for the two-type cases

Unequal access to housing, two-type model

The first order conditions for the optimization problem given in (12)-(14) with respect to \(c, H\) and \(z, i=1,4\) are

\[ N^1u_c - \lambda N^1 - \mu \hat{u}_c = 0 \]  
(30)

\[ N^1v_H \frac{1}{s^H} - \lambda N^1 - \mu \hat{v}_H \frac{1}{s^L} = 0 \]  
(31)

\[ -N^1\psi_z \frac{1}{n^L} + \lambda N^1 + \mu \psi_z \frac{1}{n^H} \]  
(32)

\[ N^4u_c - \lambda N^4 + \mu \hat{u}_c = 0 \]  
(33)

\[ N^4v_H \frac{1}{s^L} - \lambda N^4 + \mu v_H \frac{1}{s^L} = 0 \]  
(34)

\[ -N^1\psi_z \frac{1}{n^H} + \lambda N^i - \mu \psi_z \frac{1}{n^H} \]  
(35)

Endogenous housing prices, two-type model

The first order conditions for the government maximizing (12) subject to (13) and (21) with respect to \(c, H\) and \(z, i=1,4\) are given by

\[ N^1u_c - \lambda N^1 - \mu \hat{u}_c = 0 \]  
(36)

\[ N^1v_H \frac{1}{s^H} - \lambda N^1 - \mu \hat{v}_H \left( \frac{\Omega}{s^H} + \frac{H^1 d\Omega}{dH^1} \right) = 0 \]  
(37)
\[-N^1\psi_z \frac{1}{nL} + \lambda N^1 + \mu \hat{\psi}_z \frac{1}{nH}\] (38)

\[N^4 u_c - \lambda N^4 + \mu u_c = 0\] (39)

\[N^4 vH \frac{1}{sL} - \lambda N^4 + \mu vH \frac{1}{sL} - \mu \hat{v}H \frac{H^1}{sH} \frac{d\Omega}{dH^4} = 0\] (40)

\[-N^1\psi_z \frac{1}{nH} + \lambda N^1 - \mu \psi_z \frac{1}{nH}\] (41)

**Paternalism and housing, two-type model**

The first order conditions for the government maximizing (25) subject to (6) and (26) with respect to $c$ and $z$ are the same as in the general case, given by (30), (32) for type 1 and (33) and (35) for type 4. The conditions with respect to $H$ are, however, changed to

\[N^1 P_H \frac{1}{sH} - \lambda N^1 - \mu \hat{v}H \frac{1}{sL} = 0\] (42)

\[N^4 P_H \frac{1}{sL} - \lambda N^1 + \mu vH \frac{1}{sL} = 0\] (43)