TOO BIG TO RISK
Risk profile in different sized mutual funds

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The Finnish mutual fund market has multiple options for fund investors. There is a massive scale in terms of different sized mutual funds to choose from. Most of the funds are UCITS regulated equity funds and registered in Luxembourg. This research aims to reveal any correlations between the underlying risk profile of the equity based mutual funds and the size measured in assets under management. The timing is perfect for the research since the Finnish mutual fund market is celebrating its 30-year anniversary and the assets under management are at the all-time high. Additionally, the largest regulatory outline so far aiming for investor protection, MiFID II, came into force in the beginning of the year 2018. Therefore, the transparency regarding financial risks of the mutual funds cannot be highlighted enough.

The research is executed with quantitative methods and especially Pearson’s correlation tests. The examined sample is based on Morningstar Inc’s calculations and reference data. In total, the reference data consist of 1078 mutual funds with their measurements based on the availability. There are distinct capital asset pricing model-based risk measures and other relevant measures examined one at a time. Also, all measures are examined with the full sample and in deciles in order to segregate phenomenon across the full scale and possible local size ranges.

The empirical research reveals that the risk profile does not correlate remarkably with the assets under management. Though, some inconsistencies can be found from the larger end of the scale. There is indicative evidence that the diseconomies of scale are affecting the larger end of the scale. This leads to unreasonable risk taking for the investors without corresponding reward. Furthermore, the research finds that the consensus of the negative correlation between mutual funds’ expense ratios and assets under management remains. However, there are surprising inconsistencies in the mid-sized mutual funds in contrary with the consensus.
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1 INTRODUCTION

1.1 Background

Individual investors have nowadays incredible selection of investment products to choose from. There are lots of distinct asset classes and combinations of them available in the market and new investment products appear on a weekly basis. At the same, regulation is driving the protection of investors more secure and transparent way. One of the biggest acts towards transparency and investor protection, MiFID II, came into force on 3rd of January 2018 within European Union area. We could say that these are positive things taking into account that the investing in general has reached the largest scale ever measured by indices and assets under management during the asset management history.

One category of the traditional combinations of different securities and asset classes is mutual funds. Mutual funds narrow down the difference between large institutional investors and small private investors by allowing the same economies of scale available for everyone. Regardless of the amount to be invested, an investor can achieve the full diversification benefits by investing in mutual funds (Redman, Gullett et al. 2000). Mutual funds are managed by mutual fund companies, banks and portfolio managers. They are doing the asset managing on behalf of the mutual fund and on behalf of the investors accordingly with the investment strategy and rules set for the mutual fund. Mutual funds are owned by individual unit holders based on the share of the total units.

The investment fund market itself is huge. According to Investment Company Institute, the total size of the open-ended mutual fund market was 44.78 Trillion of US Dollars by the end of the second quarter of 2017. If all the funds of funds’ assets are added the amount would be 48.31 Trillion of US Dollars of total assets under management of regulated open-end funds globally. In addition to mutual funds, this figure includes 4,001 Billion of US dollars of exchange-traded funds and 4,002 Billion of US dollars of Institutional funds. (Investment Company Institute 2017b)

In Finland, the first legislation for investment funds was established on 8th of May in 1987 (Act on Common Funds 480/1987 2017). Hence, the Finnish fund industry celebrated its 30-year anniversary last year. During the history of the industry its popularity has been increasing over
the time and the offering has broaden accordingly along with the demand. It is obvious that the trend has increased the total assets under management which can be seen in figure 1 by fund types (Bank of Finland 2017b). There can be seen also, that the popularity of more complex products has been increasing and it has created even new types of investment funds e.g. real estate funds. Currently, the global fund industry is at the all-time high as it is in Finland. Obviously, it cannot be said for sure whether this trend is going to change or not and when. Though, the market data is easily available for analyzing and estimates about the direction can be made.

These conditions provide a meaningful framework also for researches focused on mutual funds. This way we are getting more academic information to support investment decisions and making the financial industry even more transparent for general public. This is a reasonable course, as investors have the rights to be aware of in which they are subscribing and investing their capital in.

Figure 1: Net assets of funds by fund type and total net subscriptions
1.2 Overview of the research

This research is intended to examine whether the size of mutual funds is affecting the risk profile and the investment decisions made by fund managers or not. This leads to an understanding of how the mutual fund shell around the investment portfolio is directing the outcome in terms of efficient market hypothesis theory. Risk-driven perspective is a good addition to the scientific field of study, as most of the studies are concentrated on the performance point of view.

This research is accomplished by measuring the correlations between the total assets under management and risk measures based on Markowitz’s modern portfolio theory and capital asset pricing -model. The research has a potential of revealing trends or correlations in the mutual fund market concerning risk profile. There is also chances to reveal a bias in the mutual fund fundamentals or other kinds of general phenomenon in certain size group of mutual funds.

The research problem is divided into the main problem and side problems accordingly with the research hypothesis and statistical testing. The main research problem is:

1. Is there any correlation between the size of equity mutual fund and the risk profile of the fund?

The additional research problems are:

2. What are the coefficients of the correlations?

3. Are there correlations between the size of equity mutual fund and the other measures of the fund?

There are three main sections with subsections in this research: Theoretical background, research framework with the tests, results and conclusion. The first part of the theoretical background distinguishes the differences between different types of investment funds. It also explains the main characteristics of fund companies and the practice of investing in mutual funds globally and more specifically in Finland. For the research it is also reasonable to take a closeup look into the examined financial year for different types of funds. This is done first globally and then on the territory level of Finnish mutual fund market. Furthermore, the theory gives an overview of the predominant regulatory environment and risk management processes for mutual funds.
The second part of the theoretical background explains the relating financial theory regarding to portfolios and surrounding shell of the mutual fund. Markowitz’s modern portfolio theory and capital asset pricing model are the most important base for the financial theory of the research. Therefore, the concept of the theory is explained with the most essential criticism related to it. Furthermore, there are multiple measures used in the examination and testing based on the previously mentioned theory which are explained and shown how they are calculated at the equation level.

Additionally, with the basic modern portfolio theory and its derivatives this research conducts also behavioral finance to explain possible differentiations from the assumed efficient market hypothesis. Since the examination and the measuring is done based on the modern portfolio theory, the framework of behavioral finance is introduced and later on used mainly as an explanatory theory for deviations from the modern portfolio theory. Regarding behavioral finance theory, there are also some mutual fund specific phenomenon which are introduced bellow the heading of behavioral finance.

![Theoretical framework](image)

*Figure 2: Theoretical framework*

To illustrate the whole of the theoretical framework there can be imagined a universe of distinct aspects which are affecting the risk profile and decision making of a mutual fund. These aspects
are financial market, mutual fund shell as an organization, current regulation and legislation, the modern portfolio theory and the behavioral finance. All of these can be seen affecting mutual funds, but it is difficult to distinguish between the volumes of different aspects. The theoretical framework is visualized in figure 2 to gain better understanding of the surrounding universe.

The second section of the research concentrates on the actual empirical research execution and statistical testing. The first part of the empirical section of the study introduces the most relevant previous researches around the same topics. These are categorized by the approach of the research and they have numerous overlapping details with this particular research. There are three main approaches categorized: size, performance and management. The findings are mirrored later with this research and evaluated whether the discoveries are aligned or they are in contrary with this research.

The second part of the second section introduces all the key components of the actual testing of the research. The most important thing of this part is to explain all the decisions made in order to end up in assumptions with the used sample. These decisions include all the limitations and prefixes which have been done to the reference data and the narrowed research sample. Additionally, these decisions include the actual methodology process that is selected for this particular research framework and setup. This leads to a clear understanding of how the research ends up with certain results and conclusions. This part also highlights the features of the used test methods to understand the pros and cons related to them. Finally, the research hypothesis are set for the tests and the actual tests are executed.

The final section concentrates on the outcome of the statistical tests. All the numeric results with the used significance levels are explained and summed up to make reasonable insights of the result data. Firstly, there are descriptive analysis of the sample from each of the examined point of views. Secondly, there are processing of the actual correlation matrixes and the values. Finally, the results are explained across all the statistically tests used and further on concluded with the theoretical background and previous research. The conclusion focuses also to gather all the essential findings with explanations and to provide directions for further research to fill the gaps which are left behind from this research.
2 THEORETICAL FRAMEWORK

2.1 Investment funds

Investment funds are investment companies and more specifically financial intermediaries investing on behalf of their customers’ or unitholders’ wealth into various kinds of assets and securities. The main idea and the key benefit of investing in investment funds are the economies of scale. Even with a little amount of money an investor may have the same benefits than a superior institutional professional investor. There are four key benefits which are coming with the size: a comprehensive administration, record keeping, diversification, professional portfolio management and investment approach and lower transaction costs. (Bodie, Kane et al. 2012, 85)

It can be seen, that investment funds differ from the other offering of banks and insurances. Investment funds are not operating directly from their own balance sheets making profits like insurances and loans. All the added value generated goes back to the individual investors excluding different kinds of underlying fees. (Müller, Ruttiens 2013. 5)

There are two types of managed investment companies based on their liquidity and administration: Open and closed-end funds. Open-end funds are traded with their net asset value (NAV). The calculation and the trading frequency varies across the open-ended investment funds. Closed-end funds are the opposite to open-end funds. They are not redeemed or issued by their management companies. Therefore, the price of the closed-end fund can differ from the net asset value of the fund. (Bodie et al. 2012, 86)

From the risk management point of view, investment funds are hedging many traditional financial risks by their nature. Especially “self-liquidating” open-end funds are eliminating the liquidity risk when the management company acts as a counter-party for the fund transactions. Also, the fair value is frequently checked by the NAV calculation, so the fund is rarely mispriced. (Haslem 2010, 5)

Risk exposure is considered and widely spread in the investment funds. Investment funds have typically a large number of securities and assets. There is no general kind of minimum, but even the most concentrated investment funds tend to carry at least twenty investment products. In addition, investors are able to gain access to the markets that they are not accessing as individual
Investors. This concerns particularly securities outside domestic markets. Therefore, investment funds are mostly affected by systematic risk or macro economical phenomenon. (Haslem 2010, 8)

Investment funds can be categorized in many ways. The Financial Supervisory Authority of Finland (FSA) (2014) approaches investment funds from two directions: Regulation and investment type. From the regulation point of view investment funds are divided as follows:

**Investment funds**

These funds are regulated by undertakings for collective investment in transferable securities (UCITS) at European Union level and on the national level by Act on Common Funds (Mutual Funds Act). Basically, these UCITS funds are referred as mutual funds. UCITS funds are the only fund type allowed using “common funds” in their company name.

**Non-UCITS funds**

Non-UCITS funds are deviating from the regulatory framework of UCITS directive. Deviations can be related to the risk profile or diversification that they are carrying. After the ratification of the Act on Alternative Investment Fund Managers (AIFMD) in 2014, Financial Supervisory Authority of Finland is no longer separating new non-UCITS funds from the alternative investment funds.

**Alternative investment funds**

Alternative investment funds are directly regulated at European Union level in the Act on Alternative Investment Fund Managers (AIFMD). The regulation came into force in 2014 which led the relatively unregulated AIFM markets to the era of regulation. Alternative investment funds can invest nearly anything and there is highly diversified scale of different alternative investment funds. The most important rules for alternative investment funds are that they need to have authorized fund managers and a comprehensive key investor information document (KIID) to provide for the investors of the fund.

Another way of categorizing investment funds by Financial Supervisory Authority of Finland is the categorization by investment type. Hence, all investment funds can be found from one of the category types. The categorization by investment type is not that clear than regulation-based categorization due to overlapping and varying investment fund markets. However, some key categories can be identified as follows:
Equity funds

Equity funds are investing only in equities and equity-linked derivatives. Derivatives are used to adjust the level of risk and goals of the fund. The investment sector, industry allocation, currencies and these kinds of features are indicated in the rules and prospectuses for the fund including key investor information document.

Fixed-income funds

Fixed-income funds are considered as lower risk level funds. These funds are investing in interest-based investment products such as bonds, debt securities and other money market instruments. The allocation and the investment scope within short-term, medium-term and long-term maturities are varying between funds.

Balanced funds

Balanced funds are basically combinations of the previous fund types. By adjusting the ratio of equity and fixed-income the fund is able to obtain the desired risk-profile. That way the riskiness of balanced funds settles between equity funds and fixed-income funds based on the fund rules. This can be seen as a component that fulfills the demands of those investors that have a risk tolerance somewhere between those distinct asset classes.

Funds of funds

Funds of funds are collateralized funds from other funds. The structure gets easily complicated for the investor, so the investor should pay attention especially to the fee structures when selecting funds of funds. From the regulation point of view, the fund of fund structure does not make any difference, so these funds must obey the same limits and rules than single layered investment funds.

Feeder funds

Feeder funds are investing at least 85 percent of their assets in another investment fund. Usually feeder fund structures allow fund managers to adjust alternatives of the master fund for investors. Usually, there can be one master fund and multiple feeder funds with different features added on top of the master fund.
2.1.1 Mutual funds

Mutual funds belong to the umbrella structure of investment funds and it is the common name for an open-ended investment company (Bodie et al. 2012, 88). To understand the business and the cashflows of the mutual funds, it is reasonable to take a closer look of the stakeholders of mutual funds. There are plenty of options for alternative arrangements of different stakeholders. The management company of the mutual fund can be a bank or a private fund management company. In addition, the mutual fund can be part of a larger fund family structure or it can be an independent entity. However, the core structure of the mutual funds remains the same regardless of the surrounding structure.

Berkowitz and Qiu (2003, 113) are illustrating the structure of the mutual fund in a comparison with publicly owned corporation to ease the understanding. The comparison can be seen in Figure 3. In a simple case of single mutual fund, the center of the operations is located in the management company. The management company is responsible of the administrative issues of the mutual fund. This can be compared to the management of a corporation. Both of these are responsible for the shareholders of the company. The management team is hired with contracts which are leading to operating expenses (C) incurred by administrative, legal, auditing and other activities in the mutual fund. Corporations are having similar activities and their management is hired with wage contracts (W₂).

Mutual funds can have internal or external portfolio managers for investment decisions and analysis for the mutual fund. The portfolio management might be a group of professionals or an individual person. These managers can be seen as employees of the management company where management company is paying wage (W) for portfolio managers. Usually this wage is more or less performance based. In a comparison, corporations have all kinds of employees with different kind of salary contracts (W₁).

The last entity in the mutual fund structure is the net value of the assets (NAV) and the total content of the fund itself. This can be compared with revenues (R) of a corporation. The NAV of the fund is divided into two parts: The fixed expense ratio (m) combining all the management fees related to the fund and the rest of the fund’s value. Management fees belong to the management company and the rest stays with the unitholders. Therefore, the value for the unitholders equals (1-m) x NAV and for the management company it is m x NAV. Furthermore, the shareholders of the management company are receiving m x NAV less all the incurring
aggregated expenses of $W - C$. With corporations the structure is slightly simpler in this sense, where all the revenues are flowing to the management and from there back to the operational expenses and administrative costs, employees and the remaining to the shareholders as a dividend. As an equation for the shareholders the combination means simply $R - W_1 - W_2$.

The payout of the mutual funds varies across mutual fund unit series. The mutual fund can have either distribution units or accumulation units. Distribution units are paying out dividends and capital distributions related to the underlying assets based on the fund’s rules. Accumulation units are reinvesting all the capital distributions and dividends, so they are not paying any other returns for unitholders but capital gains. One mutual fund can have both of these to attract more investors and the unit series can have different kinds of fee structures. The choice between these unit series is usually made accordingly with investors’ individual tax preferences or investment goals. (Financial Supervisory Authority of Finland 2014)
Figure 3: (a) Mutual fund (b) Corporation
2.1.2 Investment fund market

Global market

The mutual fund investing has been rising all the way since the second world war (Hull 2012, 71). It is by far the dominant investment company type among all the others in U.S, where it has 90% share of all investment companies (Bodie et al. 2012, 88). According to the Investment Company Institute (ICI), United States represents 54% of the total mutual fund market, so it is a good indicator of the whole world. When the investment fund market is broken down by fund types, the dominant fund type is equity based mutual funds. The equity fund market is the same sized than bond and mixed fund market combined. Other kinds of funds i.e. money market, protected, real estate and others, represent slightly over one fifth of the total fund market. This is demonstrated in Figure 4.

![Investment Fund Types Q2/2017 (ICI)](image)

Figure 4: Investment Fund Types Q2/2017 (ICI)

Africa, where the fund market is focused mostly in South Africa, is the opposite to the U.S market where is only 0,39% of the total net assets. From between, Europe is the most diversified when it comes to the investment fund types. The share of equity-based mutual funds is far more
balanced within other investment fund types and there are even more real estate funds than in U.S. measured in absolute figures. This can be seen in figure 5. (Investment Company Institute 2017a)

In comparison between United States and European investment market there can be seen a major difference between the average size of an investment fund. In U.S the average fund size settled at 1210 million euros at the end of the first quarter of 2012 while in Europe the average fund size was only 163 million euros. This also emphasizes the more diversified investment fund market in Europe compared to United States. (Müller, Ruttiens 2013, 10)

According to European Fund and Asset Management Association (2017), within the European UCITS fund industry the largest amount of net assets under management can be found from Luxembourg with 35,9% of the total net assets under management at the end of the third quarter in 2017. The second largest market is Ireland with 18,5% and United Kingdom as the third largest market with 12,3%. Correspondingly, the smallest fraction of the total net assets under management can found from European countries at the Eastern side. As examples, Bulgaria, Croatia, Malta and Cyprus are not holding more than one permille each. However, Cyprus recorded the fastest growth of 18% from the end of the second quarter while Switzerland’s share shrunk the fastest by 2,5%. It seems like that the European countries are heading towards the balance.
The European UCITS fund industry concludes the most of the total assets of all European investment funds. According to the difference between Quarterly Statistical Report Q3 2017 and Quarterly Statistical Report Q3 2012 by European Fund and Asset Management Association (2012) the dominant share of UCITS funds has slightly decreased within five-year period. The change is almost ten percent from 70.6% to 61.7%.

Finnish market

Finnish investment fund market was established due to the allowing legislation in 1987 (Act on Common Funds 480/1987 2017). Since then, the market has been expanding and developing. On 31st of August in 2017 the size of the investment funds in Finland was over 116 billion euros. The figure consists all UCITS and non-UCITS funds regulated by Financial Supervisory Authority of Finland. The spread between different fund management companies can be seen in figure 6. Investment funds are offered via bank driven fund management companies such as Nordea Funds Oy, SEB Rahastoyhtiö Suomi Oy and Danske Invest Rahastoyhtiö Oy and independent fund management companies such as Titanium Rahastoyhtiö Oy, Aurejärvi Varainhoito Oy and eQ Rahastoyhtiö Oy. Nordea Funds Oy dominates the competition with the market share of 40%. The following fund companies are OP-Rahastoyhtiö Oy with 20% share of the total and Danske Invest Rahastoyhtiö Oy with 10% share. Hence, the largest three fund companies are covering 70% of the total size of the Finnish investment fund market. (Financial Supervisory Authority of Finland 2017b)

From the same figure 6 can be seen the growth of the total market. The growth might be driven by positive net subscriptions or positive capital gains from the underlying investments. The growth’s magnitude during the one-year period until 31st of August in 2017 has been 12.29% which equals 12.8 Billion euros. As a comparison, benchmark indices have performed also fairly well. For example, OMX Helsinki Benchmark CAP GI, which is a typical benchmark index for investment funds focused on Finnish market, has gained 19.1% during the same period (Nasdaq OMX Nordic 2017). This particular benchmark index does not fit overall to the Finnish fund market size developing but it gives a good picture that the market has given good opportunities for fund management companies to grow their assets. Although, the growth of the assets has not been certainly the only driver for the growth of the total market. According to Bank of Finland (Bank of Finland 2017a), the monthly net subscriptions during the period have
been positive especially concerning the equity based UCITS and non-UCITS funds. Thus, the total Finnish investment fund market hasn’t performed accordingly with the presented benchmark index. In addition, the growth numbers of the total market have been deducted by the funds’ expenses.

Figure 6: Fund management companies’ market shares (FSA)

In the Finnish investment fund market, there are handful of different kinds of fund managers. However, this is not the whole truth for an individual investor due to the fact that Finland as a part of European Union opens its borders for foreign services, capital and fund companies as well. Therefore, Finnish fund management companies’ market share has been diluted by other European and especially Nordic fund management companies.
According to the reference data used in this study, from the investment funds offered in Finland and tracked in Morningstar Inc. database, the largest fund management company offering investment funds in Finland is Norwegian Skagen AS with 56 billion euros. This measure consists total assets under management in the offering. The second largest proportion of the Finnish investment fund market belongs to Swedish Handelsbanken Fonder AB with 49 billion euros. The first actual Finnish branch, Nordea Funds Oy, takes the third place with 41 billion euros as shown in figure 6.

Notable in the situation is that Handelsbanken and Nordea at the group level have branches also in Luxembourg offering investment funds to Finnish market. Handelsbanken’s Luxembourg offering covers 26 billion euros and Nordea’s covers almost 7 billion euros. Overall, investment funds registered in Finnish branches covers only 22,34% of the total amount of the offering when Luxembourg’s proportion covers 36,91%. The full allocation can be seen in figure 6. Hence, Luxembourg acts as an investment fund hub for Finland but also for many other countries as well. Luxembourg’s regulative atmosphere has allowed the financial sector to grow and attract a large number of foreign fund management companies within its territory. However, recent regulative streamlining and heavy debate about tax havens, offshore centers and shadow banking have challenged the advantages of Luxembourg as a financial hub. (Dorry 2015)

From figure 7 it can be seen, that Norway has also a significant role in the Finnish mutual fund market and it is slightly differently positioned than other jurisdictions. Despite the fact that Norway is not a member of European Union, it is part of the European Economic Area. Therefore, it is not privileged with any competitive advantages over Finnish branches for example. This is accomplished by a common supervising financial authority institution. Norway acts as a part of the board of supervisors of The European Securities and Markets Authority (ESMA) but it is not giving any votes for the actual decisions. This way Skagen AS can be the market leader without remarkably biasing the competition among the investment fund companies in Finland. (The European Securities and Market Authority 2017)
From the competition point of view the examined financial year has realized some differences between the territories for equity based mutual funds. Compared with some of the basic measures related to performance, risk and expenses it is possible to evaluate the success of different territory separately. According to the reference data from Morningstar Inc. Estonia stands out from other territories measured with alpha. However, the data of Estonia consists mostly from the mutual funds investing in Eastern Europe and Russia and nine of eleven fund entries are from Trigon Asset Management AS, so it could explain the relatively high performance compared with other territories. Comparing the largest shares of the total Finnish market, Luxembourg, Finland and Sweden, the deviation is not that clear. Luxembourg has performed with a tiny addition of success compared to Finland and Sweden measured with risk-adjusted return, Sharpe ratio. On the other hand, if the risk is not taken into account, Finland has gained the best alpha figures, which gives the best return amounts in euros. These details can be seen in figure 8. The figure is structured in descending order by Sharpe ratio.

From the expenses point of view, territories are more or less at the same line. Most of the territories are settling near 1,6 % in their yearly recurring fees. United Kingdom is lacking expense figures due to the data which is consisting mostly from Fidelity Investments’ funds which haven’t delivered their expense ratios to Morningstar Inc. Furthermore, Estonia appears
to have higher expenses, but likewise this could be an outcome of the relatively high Trigon Asset Management AS proportion.

Figure 8: Finnish mutual fund market averages by branches territory (Morningstar Inc.)

2.1.3 Regulatory framework

Since Finland is a member state of European Union, it is also part of the European system of financial supervision. The system operates bellow the guidance of European Parliament and Council. The common supervision system comprises from three separate entities: The European Securities and Markets Authorities (ESMA), The European Banking Authority (EBA) and The European Insurance and Occupational Pensions Authority (EIOPA) which are representing the micro-prudential supervision department of the system. In addition, the system comprises the macro-prudential oversight of The European Systemic Risk Board (ESRB). The structure is illustrated in graphical way in Figure 9 by Financial Supervisory Authority of Finland (FSA) (2017a). Figure demonstrates the role of the national supervisory entities as members of decision making and implementation of the complete system of the European union. The
common legislation is harmonized through the national supervisors and that way the whole system ensures appropriate, efficient and seamless financial markets within the European Economic Area. (Financial Supervisory Authority of Finland 2017a)

The most forcing regulatory framework for European Union members and other jurisdictions included in The European Economic Area for mutual fund markets is the UCITS (Undertakings in Collective Investments and Transferable Securities) directive. The current major version of the directive is UCITS IV which entered into force in July 2012 (Financial Supervisory Authority of Finland 2014). According to the press release of Financial Supervisory Authority of Finland (2010) the regulation was intended to enter force already one year before in July 2011, but for massive scale regulatory renewals it is typical to be postponed. There is also UCITS V directive under implementation process and it concerns mostly changes in UCITS IV in depositary, salary, fees and sanction policies (The Ministry of Finance 2017).

The main purpose for the cross-border regulation is to facilitate cross-border fund distributions and improve the protection of individual fund investors. It allows fund management companies to distribute their funds via different branches in different member states. This is done by Management Company Passport (MCP) which also tightens the requirements of the organized risk management procedures within the fund management company. Cross-border mergers are also eased by the UCITS IV framework among with more collaboration between the financial
competition authorities of the member states. (Financial Supervisory Authority of Finland 2010)

Furthermore, UCITS IV requires fund companies to provide more structured fund prospectus, Key Investor Information Document (KIID), for the investors. The prospectus eases the comparing of different funds and clarifies the main features of the fund in simple and understandable way. It specifies the investment policy and objectives, risk-reward measures, fees and past performance. That is the last resort, where the investor gets the idea of which kind of investment product he is about to invest in in terms of complexity. It is the only document which needs to be provided in the member state’s language. Hence, Key Investor Information Document plays significantly important role in investor protection. (Financial Supervisory Authority of Finland 2010)

In Finland, investment funds are also dependent of other institutions and legislation. Investment funds are typically distributed via investment advisors and sales channels by different asset managers. Therefore, regulation concerning investment advising and sales concerns also investment funds. The biggest regulation concerning investor protection and financial product distribution was MiFID regulation due to the credit crisis initiated in 2007 and the development of the markets. Later on, the regulation was decided to be renewed with MiFID II and MiFIR regulations. It was seen, that the old MiFID regulation was outdated in terms of investment products, processes and accuracy. The authorities demanded more details about the investment products and the underlying fees in order to strengthen the markets against systemic risks and increase investor protection. However, this new regulation framework ended up so broad and comprehensive, that is was forced to be postponed by the European parliament. Eventually, the regulation came into force at the beginning of the year 2018. (Financial Supervisory Authority of Finland 2017c)

Bank of Finland acts also as an important institution for investment funds. The most essential tasks for Bank of Finland are preparing and implementing monetary policy, conducting research and statistics of the economy and maintaining stability and efficiency of the payment system within the jurisdiction. These tasks are accomplished in collaboration with Financial Supervisory Authority, European Central Bank and Statistic Finland. (Bank of Finland 2017c)

Finnish investment funds are providing information about their operations for Bank of Finland. Bank of Finland is collecting information on a monthly basis about investment funds’ balance sheets, subscriptions and redemptions. The data is collected via electronic format of SIRA
survey. The information is onwards forwarded to Statistics Finland and European Central Bank. It is used for tasks such as statistics, monetary policy and macro prudential oversight. (Bank of Finland 2010)

### 2.1.4 Risk management

Risk management of traditional mutual funds operating in Finland have built-in risk management processes. The reason for that is driven and required by UCITS regulation framework. In general, the UCITS directive outlines the risk management processes as follows:

“A management or investment company shall employ a risk-management process which enables it to monitor and measure at any time the risk of the positions and their contribution to the overall risk profile of the portfolio.

It shall employ a process for accurate and independent assessment of the value of OTC derivatives.

It shall communicate to the competent authorities of its home Member State regularly in regard to the types of derivative instruments, the underlying risks, the quantitative limits and the methods which are chosen in order to estimate the risks associated with transactions in derivative instruments regarding each managed UCITS.”


Furthermore, the Committee of European Securities Regulators (CESR) has published Guidelines on Risk Management outlining the requirements for risk management in UCITS regulated mutual funds. The issue concludes guidelines for risk management and compliance operations within the organization and the actual categorization of the relevant risks for UCITS funds. (The Committee of European Securities Regulators 2009)

The most immediate rule set for UCITS funds concerns their investment limits. These limits are fully applicable after six months from the inception of a new UCITS fund. The limits are set for issuer, issuer group and portfolio diversification. This way the fund is hedged against overwhelming dependence of another counter parties or exposures. A UCITS fund shall not invest more than 10% of its assets by one issuer. The limit is broader when the issuer is a
sovereign issuer e.g. member state of the European Union or when the issuer is issuing covered bonds. There are also some differences between jurisdictions for example Luxembourg regulator CSSF (Commission de Surveillance du Secteur Financier) which allows 100% weight on a single sovereign issuing counterparty. The limit is also increased to 20% with index linked funds. Furthermore, 20% is also a limit to the same issuing group structure. (Müller, Ruttiens 2013, 22)

There is a 10% limit on assets invested in other funds which prevents UCITS fund to evade regulatory rule sets via another investment fund. At the security level, UCITS rules allows investing in different security types but the underlying securities must be traded on a regulated market and, especially concerning structured products, the maximum loss must be limited to the purchase value of the security. Also, unlisted and private equity securities shall not exceed the limit of 10% of the total value of the fund. In addition, there are also more detailed limits concerning over-the-counter products and money market instruments. (Müller, Ruttiens 2013, 21)

It is obvious, that portfolio managers shall not exceed these limits by purpose. However, it is usual that the market fluctuations are causing passive breaches to these limits. In that case, the portfolio manager must rebalance the portfolio the way that is aligned with the investor’s interest. For example, this could mean that the portfolio manager should not sell directly the exceeding security if the transaction is causing immediate losses to the fund. In case of any active breaches by portfolio managers, the fund administration must take actions to prevent these breaches in the future and notify the financial competitive authority about the event. (Müller, Ruttiens 2013, 23)

Risk management wise, UCITS directive outlines also mutual funds to use appropriate and efficient portfolio management techniques. These techniques should be also presented in the fund prospectus or Key Investor Information Document and they need to be followed precisely. The efficient portfolio management techniques conclude practices related to corporate actions of the securities e.g. entering a repurchase program without exposing the fund to an unsustainable risk compared to the fund prospectus and investor’s interest. The liquidity risk must be also taken into account with efficient portfolio management techniques especially concerning the ability to execute the pending fund redemptions at any given time promised in the fund prospectus. Poor liquidity risk management within an open-ended mutual fund could easily lead in the situation where the fund is unable to pay out the redemptions from the illiquid
investments if it is not allocating enough cash. (The European Securities and Market Authority 2014, 7)

In accordance to the quantitative point of view there are multiple quantitative ways of measuring risk. According to Longin (2017, 216), standard deviation of the returns and beta can be described as traditional measures of risk. These measures are used also in this research and introduced later more precisely. There are also more extensive ways of analyzing financial risk which are broadly in use within investment banking and reinsurance business. The background of multiple quantitative risk measuring techniques belong under the umbrella of the extreme value theory which broke the old tendency of categorizing abnormal events as outliers of the sample in statistical researches. One broadly used application of extreme value theory-based risk measures are tail risk measures such as value at risk (VaR), conditional value at risk (CVaR) and expected shortfall. Value at risk measures the possible fall in the worst-case scenario in terms of relative share of the portfolio. Though, there are deviations among institutions whether they are using significance level of 1% or 5%. For example, within investment banks Goldman Sachs utilizes 5%-VaR where Citigroup, Credit Suisse First Boston, Deutsche Bank, JP Morgan Chase and Morgan Stanley are utilizing 1%-VaR.

Overall, mutual fund risk management is a broad combination of investor protection, systemic risk mitigating and prevention of abusing voting power or whatsoever. However, there is still space for mutual funds’ administrations to evaluate the suitable risk management procedures for mutual funds. Therefore, most likely mutual funds are not using the exact same risk measurement techniques and figures even within the same fund type while analyzing the different risk exposures of the mutual fund portfolio.

For all the mutual fund organizations it is common that enterprise level risk management procedures can be applied to fund administration and mutual fund organizations to mitigate all the risk categories in organized and consistent way. Applying the research of Gatzert, Schmeiser et al. (2008, 256), especially for fund families it is important to acknowledge the risk concentrations and the total level of diversification among the fund family regardless the fact that not all the stakeholders are requiring diversification. In addition, all dependencies among the distinct entities of the mutual fund family should be mapped to evaluate enterprise risk management of the group. Otherwise, regardless that the total risk is diversified evenly within the fund family, there can be generated systemic risk which could ignite a harmful domino effect causing losses.
The enterprise risk management procedures in mutual funds are typically handled through the compliance department of the administration of the mutual fund. According to Ramakrishna (2015, 41), financial markets are the driver of regulation and regulation is the driver of compliance. Hence, compliance is a sub-category for regulation but it is located inside the organization of a financial company such as mutual fund meaning that the compliance establishes the guidelines for the organization in terms of implementing regulation and legislation, industry standards and code of conducts. Thus, compliance is the responsible for external requirements and also internal enterprise risk management procedures regarding operational risks, market risks, liquidity risks, credit risks and other risks.
2.2 Modern portfolio theory

In 1950-century, Harry Markowitz came up with his study, Portfolio Selection, and he was awarded with a Nobel Prize in Economics later on in 1990. Markowitz has a broad background with programming at UCLA University, RAND Corporation and General Electrics. He was also experienced within wealth management when he was working as a CEO for hedge fund named Arbitrage Management Company. (Malkiel 1999, 192)

Harry Markowitz aimed to answer perpetual questions for the relationship between risk and return. This trade-off assumes that the greater the risk taken is, the higher the return can be realized. This trade-off is usually misleadingly shortened as a trade-off between risk and return, when it should be a trade-off between risk and expected return. Expected returns are basically weighted averages of possible returns based on their historical data. (Hull 2012, 2)

The biggest finding in the study, Portfolio Selection, was that the saying: “Don’t put all your eggs in one basket”, applies especially to the financial market. This theory is called modern portfolio theory and it gives the framework to combine financial risk and return with investment portfolios. This theory shows how investing in multiple securities diversify the total risk and reduces the exposure of company-specific factors or in other words, non-systematic risk. This effect is demonstrated in figure 10. (Bodie et al. 2012, 149)

![Figure 10: Systematic and non-systematic risk](image)

*Figure 10: Systematic and non-systematic risk*
The modern portfolio theory assumes the financial markets to be efficient. The efficient market hypothesis assumes that the market is too efficient to gain consistent excess returns without exposing the investments to greater risk at the same time. The market is assumed to follow so called “random walk”. When simplified, this assumption dilutes the base of the traditional stock-picking strategies and other active trading systems. However, this theory is traditionally at least partially ignored and the belief in active asset and portfolio management is maintained. It is easy to believe, that the chance to outperform the market increases with active portfolio management regardless of the academic evidence. (Haslem 2010, 95)

The base for active asset managing comes from the different opinions about the market efficiency. The market efficiency can be seen based on the point of view about the reflected information in the market prices. Economist have defined three levels of market efficiency: weak, semi-strong and strong market efficiency. Weak form assumes that the market prices are reflecting all the information from the history. Therefore, it is possible to make assumptions about the upcoming price changes by comprehensively researching the public information about the security and the surrounding market. Semi-strong market efficiency assumes that this public information is already reflected in the market prices, so there is no way to maintain excess returns with past and current public information about the stock. In practice, it would mean that the public announcements by stock companies are immediately applied in the prices when the announcement is given. Strong market efficiency takes the point of view to the furthest. It assumes that the market prices are already reflecting information that is not even announced yet. This would mean that all public, past and insider information is applied to the prices constantly, so asset management is just a matter of luck. (Brealey, Myers et al. 2012, 318)

The modern portfolio theory inspired also other researchers to look for the equilibrium between diversified uncertainty and total returns. Karl Borch (1962) applied risk-reward-ratio concept into global commodity markets from the reinsurance perspective to evaluate the suitability of the concept. He found that the commodity market as a whole affects individual negotiation which affects the decisions of rational people under uncertainty. Therefore, further research had to be applied to determine common market price more accurately. Eventually, capital asset pricing model was discovered.
2.2.1 Capital Asset Pricing Model

Markowitz did not provide answers for the relationship or the ratio between portfolio risk and systematic risk. Systematic risk is all the risk taken from the market which cannot be reduced by diversification. Even if the portfolio is widely diversified over different kinds of assets, the stock market as a whole adds the volatility to the portfolio regardless of the diversification. William F. Sharpe continued in 1964 from where Markowitz left and developed what is known as capital asset pricing model which combines expected return and systematic risk. (Hull 2012, 2)

The key finding in the capital asset pricing model is concluded in the following equation:

\[
E(R) = R_F + \beta [E(R_M) - R_F]
\]

\(E(R)\) = Total expected return
\(E(R_M)\) = Expected return of the market portfolio
\(R_F\) = Risk-free rate
\(\beta\) = Beta

In overall, the equation states, that the total expected return is divided in two: Risk-free rate and the excess expected return over the risk-free rate. When the excess expected return over the risk-free rate is zero, for example in the case where has been invested only in treasury bills, beta is zero and the total expected return equals risk-free rate. (Malkiel 1999, 209)

Capital asset pricing model can be shown in a graphical way as in figure 11. This highlights the linear form of the equation where the total expected return grows hand in hand with portfolio beta.
2.2.2 Criticism about the capital asset pricing model

The capital asset pricing model and beta was the common way of measuring portfolio risk during the 1970 century. Beta was even accepted to be an official risk measure by United States Securities and Exchange Commission. In general, it was used for multiple investment strategies as a dominant factor of the strategy. (Malkiel 1999, 210)

However, in 1992 Eugene Fama and Kenneth French published their research, The Cross-Section of Expected Stock Returns, where the capital asset pricing model and beta were questioned. The findings stated that linear regression between total expected return and systematic risk cannot be found according to their empirical analysis based on data from 1963 to 1990. (Fama, French 1992)

Hence, beta was announced to be dead in public media. Headlines were screaming that “The Death of Beta” and “Beta beaten” as examples. However, it is not completely justified to make such assumptions that beta couldn’t explain the risk of securities at all. Firstly, Professor Richard Roll from the University of California, Los Angeles, has stated that it is almost impossible to measure beta fully due to the scale and complexity of total market. To measure the correct and complete beta in addition to equities there should be taken into account things such as bonds, real estate, commodities, structured combinations of these and even mental capital.
e.g. education. Secondly, another argument for beta is the dependency of market’s definition. With different definitions it is possible to achieve different values of beta. Finally, there are also researches done with longer time-horizon that the one used by Fama and French where the key finding was positive correlation between equity beta and returns. Nevertheless, beta cannot be totally abandoned because it has proven to be reliable measure in empirical experience. If beta was irrelevant, why would it be reasonable to start running an oil rig business if it is possible to get the same returns with T-bills? (Malkiel 1999, 214)
2.3 Measures

Risk and volatility

The risks within financial markets are driven by fluctuations of the returns. The modern portfolio theory showed that there are two kinds of risk: company specific risk or non-systematic risk and market risk or systematic risk. Therefore, the total risk of a specific investment is a sum of systematic and non-systematic price fluctuations. (Treynor 2008, 6)

Risk in the financial markets is usually referred to statistical sciences’ measures of spread. The main measures for risk are the statistical uncertainty measures, variance and standard deviation. Variance is a numeric measure to track the portfolio return deviations of the expected returns. Standard deviation is simply the square root of variance. (Brealey et al. 2012, 163)

\[
\sigma^2 = \frac{\sum_{t=1}^{n} [R_p - E(R_p)]^2}{n-1}
\]

\(\sigma\) = Standard deviation (Volatility)

\(\sigma^2\) = Variance

n = Number of return periods

\(R_p\) = Return of the portfolio

\(E(R_p)\) = Total expected return of the portfolio

In financial literature and markets standard deviation of the returns are often referred as volatility. To distinguish the difference between standard deviation and volatility, we need to define a period of time. Volatility is the standard deviation of returns gained by the variable per unit of time when returns are expressed in continuous compounding. In terms of risk management, volatility is usually referred as standard deviation of the continuously compounded return per one day. (Hull 2012, 201)

If the total aggregated measures are unknown, it is possible to calculate the total portfolio variance from individual securities of the portfolio. Portfolio variance can be calculated with
the equation below. If there are more securities than two, all security pairs are added into the equation accordingly. (Brealey et al. 2012, 171)

\[ \sigma_P^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2(x_1 x_2 \rho_{12} \sigma_1 \sigma_2) \]

\( \sigma_P \) = Portfolio’s variance  
\( \rho \) = Correlation coefficient  
\( x \) = Security’s weight in the portfolio

**Beta**

Beta is defined as a sensitivity of the funds’ returns to the market movements. It measures the excess returns of the funds over the benchmark index and T-bills. More specifically, it tells how much extra the price moves for each 1% change in the market index. (Bodie et al. 2012, 308)

Beta is one part of the CAPM equation and it is the coefficient factor of the market portfolio. Thus, beta is a measure of systematic risk, so it is not explaining the total volatility of the portfolio. The equation below is proved from the equation of Jensen’s alpha which is introduced after.

\[ \beta_P = \frac{E(R_P) - \alpha_P}{E(R_M)} \]

Also, regarding to Jensen’s alpha, Morningstar Inc. is calculating betas of the funds with the same linear regression analysis than Jensen’s alphas. (Haslem 2010, 167)
Jensen’s alpha

Jensen’s alpha or simply alpha of a portfolio is a measure for abnormal returns. Alpha explains how much on average the portfolio price changes when the market index keeps stable (Bodie et al. 2012, 308). Alpha is one of the most compared measures within mutual funds because it tells which funds succeeded to outperform the market and by how much. It can be concluded that alpha tells the skill of fund managers (Bodie et al. 2012, 609).

The equation is almost the same than the capital asset pricing model’s but alpha is a non-systematic component added to represent excess or abnormal returns over the market portfolio. This is because the CAPM hypothesis by itself assumes the market portfolio to be mean-variance efficient. (Bodie et al. 2012, 599)

\[
\alpha_P = E(R_P) - \beta_P E(R_M)
\]

\(\alpha_P\) = Jensen’s alpha

\(E(R_P)\) = Total expected return of the portfolio

\(\beta_P\) = Beta of the portfolio

\(E(R_M)\) = Total expected return of the market portfolio

Morningstar Inc. is calculating Jensen’s alphas for funds by calculating the difference between the category benchmark and the fund’s excess return adjusted with 90-day U.S. Treasury bill in a linear regression analysis. Morningstar Inc. selects benchmarks for all the funds by their category group. (Haslem 2010, 166)

Sharpe ratio

The risk-reward ratio was firstly introduced by A. D. Roy to measure strategy performance. Further on, Sharpe ratio was created by William F. Sharpe likewise capital asset pricing model based on the modern portfolio theory (Bailey, de Prado 2012, 3). Sharpe ratio measures the
equation called reward-to-variability ratio (Sharpe 1964). The equation is the division of average excess return over risk free rate and the volatility of certain period.

\[ S = \frac{R_P - R_F}{\sigma_P} \]

\( S \) = Sharpe ratio
\( R_P \) = Return of the portfolio
\( R_F \) = Risk-free rate
\( \sigma_P \) = Volatility of the portfolio

Sharpe ratio is a measure for tracking risk-adjusted performance of investment managers (Brealey et al. 2012, 191). Sharpe ratio equalizes the portfolios’ to be relatively comparable with risk-free alternative. This way it is a great measure for comparison between two different mutual funds with different levels of volatility. (Haslem 2010, 166)

Sharpe ratio is criticized by its biased statistical traits. There can be found non-normality in distributions and reduced granularity. This is why some economists are saying Sharpe ratio tends to be exaggerative (Bailey, de Prado 2012). Also, Sharpe ratio is not taking into account the difference between systematic and non-systematic risk. Volatility is calculated regardless of the sources. However, Sharpe ratio is accurate enough to be highly popular and widely in use for portfolio’s performance tracking. Sharpe ratio’s benefit is also its simplicity and easy understandability.

Morningstar Inc. is calculating Sharpe ratio with 90-day U.S. Treasury bill as a risk-free rate for the monthly excess return. For volatility, annualized standard deviation is used as divisor for annualized excess return. Morningstar’s Sharpe ratio is first calculated on a monthly basis and then annualized by using the one-month Sharpe ratio. (Haslem 2010, 166)
**Treynor ratio**

JL Treynor (1965) introduced his theory which states that when the residual, company specific risk can be diversified away from the portfolio, portfolio’s return should be compared against systematic market risk. Therefore, Treynor ratio or Treynor’s measure distinguishes the systematic risk from the active portfolio risk. Treynor ratio equation differs from Sharpe ratio by risk against which the return is compared.

\[
T = \frac{R_P - R_F}{\beta_P}
\]

(7)

\(T = \) Treynor ratio

\(R_P = \) Return of the portfolio

\(R_F = \) Risk-free rate

\(\beta_P = \) Beta of the portfolio

Initially Treynor defined \(R_F\) to be index rate, but later on it has been calculated with the risk-free rate. There should be also taken into account, that Treynor ratio is not a suitable risk measure for investment vehicles which are not affected by the financial market or systematic risk in general. In that case beta of the equation is closing zero and the value of the Treynor ratio increases to perpetuity. (Hübner 2005)

**Tracking error**

Tracking error is an important measure when it comes to the difference between actively managed mutual funds and index linked mutual funds. It answers the question: How accurately does the investment fund track the benchmark index? (Hull 2012, 73)

Tracking error is defined as the volatility of return differences between a portfolio and the benchmark index (Pope, Yadav 1994, 27).
(8) \[ TE = \sqrt{\frac{\sum_{i=1}^{n}(R_P - R_B)^2}{n - 1}} \]

TE = Tracking error

n = Number of return periods

R_P = Return of the portfolio

R_B = Return of the benchmark index

Tracking error is also important for rebalancing strategies, because portfolios tend to drift towards riskier assets. Hence, when the tracking error declines too high, the portfolio manager executes the rebalancing process. That way for index fund’s compliance department tracking error provides a good tool for accurate measuring of the strategy. (Chan, Ramkumar 2011, 54)

For individual fund investors, tracking error is a good tool to measure the justifications of the expenses. If the tracking error appears to be low regardless that the fund strategy states the fund to be actively managed, correspondingly investors shouldn’t pay the fees of the active management. Recently the topic has been controversial in Finland, when actively managed mutual funds have exposed to be hidden index funds. This type of fund management can be compared to be a fraud.

Information ratio

Information ratio is important especially for active fund managers to measure the average excess return to active risk (Gupta, Prajogi et al. 1999). The equation concludes ratio between the remainder of the difference of the funds return to benchmark’s return and the tracking error of the portfolio.
IR = Information ratio

*R_P* = Return of the portfolio

*R_B* = Return of the benchmark index

TE = Tracking error

Treynor and Black (1973, 66) came up with Information ratio in order to distinguish the difference between actively managed portfolio and market portfolio. The ratio is answering the question: how much additional return has been created out of additional risk? Thus, it is a measure to analyze risk worthiness. If the ratio is very low, probably it wasn’t worth it to take additional risk by stock picking instead of investing in index funds or tracking indices. This highlights the value of Information ratio for the whole mutual fund and asset management industry.

*Expense ratio*

Expense ratio, or often referred as total expense ratio, combines all the concluded fees relating to the fund and are reoccurring on a yearly basis e.g. distribution, marketing costs operation costs, administrative costs and so on. Therefore, the rate of return of the fund equals the gross return on the underlying portfolio minus the total expense ratio (Bodie et al. 2012, 93).

\[
TER = \frac{\text{Total Fund Costs}}{\text{Total Fund Assets}}
\]

However, expense ratio does not include expense loads related to subscriptions or redemptions e.g. transaction fees, brokerage costs or other sales charges. Practically, expense ratio tells about how much it costs to hold a fund over one financial year (Haslem 2010, 191). During 2011, U.S. equity funds averaged 1.43\% of expense ratio but if adjusted with assets under management, average expense ratio was relatively smaller, 0.79\%. This is a result of the trend
where larger funds have relatively smaller operating costs due to the economies of scale. Expense ratios vary mostly across different fund types. (Bodie et al. 2012, 91)

However, expense ratios are not hand in hand with funds’ alphas. Funds can end up with negative alphas resulted by relatively high expense ratios if the expenses are not present in the benchmark index. This way expensive funds are pressured for higher alphas in order to maintain in the competition (Haslem 2010, 168). In addition, this phenomenon can be seen in comparison between actively and passively managed mutual funds. Expenses are raising among the complexity of the fund’s strategy. Hence, passively managed, index tracking mutual funds are relatively cheaper than multi-asset and multi-strategy hedge funds. (Bodie et al. 2012, 138)

Expenses are usually deducted from the investors on a daily basis. As an example, if the expense ratio of the fund is 1%, investors are deducted by 1% multiplied by 1/365 in each day. This way expenses are distributed equally within the investment period and daily NAV calculation. (Haslem 2010, 54)

To calculate the total unitholder costs, investor have to take into account the costs related to the subscriptions and redemptions. These costs are not included in the total expense ratio and they might represent a large share of the fees concerning the total costs of the holding. This happens especially when the holding time is relatively short. The total cost of the subscription is usually referred as the front-end load and the total cost of the redemption as a back-end load or deferred load. (Hull 2012, 74)
2.4 Mutual fund risk behavior

Behavioral finance

Mutual funds as a part of the global financial system are affected by human behavior or irrationality against the relatively straight forward modern portfolio theory. It provides alternatives to the random walk of the efficient market hypothesis. Therefore, the concept of behavioral finance is essential for understanding of the investment behavior of investors and fund managers.

The theory of behavioral finance was established by Daniel Kahneman and Amos Tversky. In the beginning, the behavioral finance was highly controversial and it took over twenty years to be accepted within economists and researchers. After Tversky’s pass away in 1996, Kahneman won Nobel prize in economics for the work which was done by both. Since then, the behavioral finance affected the studies of economics but as well all behavioral psychology studies. (Malkiel 1999, 222)

According to Pompian (2012), behavioral finance is “about understanding how people make financial decisions, both individually and collectively.” By adapting to the effects of behavioral finance, the advisors are potentially able to improve financial outcomes. According to many studies, markets are relatively rational and efficient supporting the efficient market hypothesis, but numerous, persistent anomalies can be recognized. Thus, for gaining alpha, portfolio managers should determine the right ratio between rationality and irrationality in the global market.

It is shown, that under exhaustion of information people are facing difficulties to maintain rationality. The clearest examples of behavioral finance are the financial crisis and meltdowns. We can’t say, that the markets were acting rationally during the United States’ housing bubble in the 21st century. Hence, by identifying the potential behavioral biases and threats beforehand portfolio manager can save the portfolio or a mutual fund from the financial disaster. (Pompian, ebrary 2012, 45)

There are certain explanations for investor irrationality. One factor to drive market prices to behave irrational is the complexity to valuate common stock. There are different methods and practices for the valuating and in addition it can be laborious and time consuming. Therefore,
investors may find it easier to take yesterday’s price as given which reduces the workload to examine only the changes happened today. This drives the price to slowly diverge from the original calculation method. When the price abnormality is finally noticed the market faces increased volatility after which this cascade starts all over again. (Brealey et al. 2012, 325)

One typical example of the behavioral finance was introduced by Kahneman and Tversky in 1979. They named the phenomenon as “Prospect theory” by the nature of the behavioral bias. The main idea of the theory is that people have a tendency to discard components that are shared among all prospects which leads to inconsistency compared between the same situation presented in different forms. (Kahneman, Tversky 1979, 263)

The prospect theory along with other behavioral finance phenomenon are affecting the decision making of individual investors and therefore the fund managers as well. In addition, mutual funds have typical behavioral features which are drifting the investment decisions away from the rational behavior. These features are the agency problem and the tournament behavior of mutual funds.

Agency problem

Mutual funds are legal entities which exist for making profits and maximizing the value for the unitholders or shareholders like most of the vital corporations. Therefore, the approach to the risk behavior is many ways similar compared to the other corporations. However, mutual funds are differing from ordinary corporations by their highly divided ownership. The management company of the mutual fund is owned by its shareholders and the mutual fund itself is owned by individual unitholders. This way the combination is a mixture of different incentives and risk approaches. The potential conflict between the stakeholders might lead into a classic example of an agency problem of the situation where the ownership and management are divided. (Chevalier, Ellison 1997, 1168)

To maximize unitholder value, mutual funds have a management unit or a fund company with hired managers for making operational decisions based on their expertise and incentives. The problem occurs when the unitholders are driven by maximizing risk-adjusted returns and the fund company is driven by the inflows of the unitholders and other profits (Chevalier, Ellison 1997, 1168). Managers’ contracts must be designed the way that the managers are directed
Towards the right outcomes. According to Clifford and René (1985, 399) managers are not maximizing the unitholder wealth without facing them with proper incentives. In ordinary corporation, this is done by observing provisions of managerial compensation frequently, so the managers have similar incentives to gain wealth within the company. In mutual funds, the situation is more complex due to a diverse ownership and structure.

The expected return and the total risk exposed to the mutual fund are hand in hand. The manager is contracted to gain profits, but in addition the mutual fund shouldn’t exceed reasonable risk levels. According to Clifford and René (1985, 401), the manager of an ordinary corporation will bear risk only if he is rewarded for doing so by contractually higher expected income. Thus, the manager’s contract should be in a perfect balance of the tradeoff between expected income and risk of income. That way the owners can ensure a proper hedging policy within the corporation. Likewise, with the performance point of view, the situation with mutual funds is sharing the same issue, but more complex way.

Overall, mutual funds are prone to an agency problem. This may cause biases in their performance and risk. When examining mutual funds, effects of an agency problem should be considered before making too simple assumptions.

**Mutual fund tournament**

Mutual funds are highly competitive with each other. They are competing of the investment inflows coming from the investors. The incentives of the investors are not always the same, but the main idea of investing the excess wealth remains unchangeable. Especially, in the situation where the mutual funds are having comparable investment objectives the competition among mutual funds can be seen as a tournament. Seeing the competition as a tournament provides us a better understanding of the portfolio decision-making process. (Brown, Harlow et al. 1996, 85)

The main goal of the tournament is obviously to outperform the market and obtain the best performance figures possible among the competitors. According to Huang, Wei et al. (2007, 1273) past performance of the mutual funds entitles to disproportionately large new money inflows and on the other hand poor performance leads to relatively smaller outflows. Combined with the fact that the mutual funds are gaining management fees based on the total assets under
management, winning the so-called tournament awards the mutual fund with new money inflows and direct benefits for the business. As a phenomenon, this trade-off drives mutual funds towards better performance and efficiency which can be considered beneficial for individual investors as well.

However, this phenomenon affects the risk behavior of mutual funds within the financial year due to the flow-performance relationship and the agency problem. According to Taylor and Jonathan (2003), the tournament-styled business environment tends to create incentives to fund managers to deviate from the investors’ desired portfolio choice in terms of risk. Chevalier and Ellison (1997, 1184) have modeled the phenomenon in a graphical way where can be seen the nature of the tournament. It shows, that at the end of the year the mutual funds which are behind the market tend to catch up the performance by increasing risk taking and correspondingly the mutual funds which are ahead of the market tend to lock in their gains by reducing the deviation from the index or tracking error of the portfolio. That way the current position in the tournament affects the risk approach of the competitive mutual funds.

Such phenomenon can be seen harmful for the investors. All the investment decision should be in accordance with the prospectus and investment strategy and completely transparent for the investors based on the regulatory demands such as MiFID II. However, such tournament behavior might be extremely difficult to harness with suitable regulatory requirements and especially monitoring and sanctions by the financial authorities.
3 PREVIOUS RESEARCH

Mutual funds are researched broadly especially from the performance point of view. People tend to look for better and more profitable investment opportunities outsourcing the decision making of individual stocks which points the pressure on mutual funds. Every percent matters when the scale is enlarged enough. Furthermore, risk point of view matters likewise. Investors tend to avoid additional and unnecessary risk that they are not paying for. It can be concluded that every consistent and statistically significant phenomenon which affects the flow between the capital subscribed to the mutual fund and the capital realized with profits or losses are highly interesting and meaningful for the whole industry. Moreover, it is not a miracle that there are plenty of research available about mutual funds. Mutual funds are documenting publicly their past track records and their specifications, so they are an easy target for researchers. Therefore, research provides a solid base for decision making when choosing between different kinds of mutual funds.

This research concentrates on the relationship between the size of the mutual fund and the risks measured based on the capital asset pricing model. However, for background information it is important to go through other relations with mutual funds. In this study, the most important approaches of the previous researches are presented which are the mutual fund’s size, performance and the management structure’s effect. These points of views are usually not isolated from each other but rather combinations with certain focus.

3.1 Size

Diseconomies of scale

Indro, Jiang et al. (1999) researched mutual funds broadly from the size point of view. Their main focus was in the relationship between the mutual fund size and the performance. They were exploring this by identifying optimal thresholds of assets under management to gain positive returns on the market information. They were relying on the theory conducted by Perold and Salomon (1991) which introduced the diseconomies of scale with assets under management in active fund management. The theory presents that the return-maximizing point
can be found from the spot where the cost of additional trading exceeds the opportunity cost of not trading. Even though the nominal returns are increasing after the optimal point, the relative returns start to decline. This happens due to increasing relative transaction costs in a concave relation with the executed block size. Therefore, the mutual fund needs to maintain within the optimal size in assets under management in order to stay in the competition with other mutual funds. This theory can be seen interesting because it breaks the belief that economics of scale are always working when it comes to investing.

The research conducted by Indro, Jiang et al. (1999) was accomplished with a sample of 683 U.S actively managed and nonindexed equity mutual funds which were obtained from the Morningstar Inc.’s database. The mutual funds were categorized accordingly with their investment style such as value, growth or blend. They were also categorized by assets under management into ten equal group measured by amount of funds. The statistical analysis consisted variables such as fund size, three-year average return, beta, residual, price per equity-ratio (P/E), price per book-ratio (P/B), median market cap, expense ratio and turnover.

The research flipped over the earlier research which had passed the relation between the efficiency of an active asset management strategy and the size of the mutual fund. The conclusion showed that there are two significant thresholds for mutual funds: minimum and maximum. Mutual funds need to reach a minimum fund size before they can fully benefit of the economics of scale in trading and administrative costs. Maximum threshold has been reached when the mutual funds start to suffer from the diseconomies of scale which were introduced by Perold and Salomon (1991) including higher transaction costs, administrative stress, deviations from the investment style, inefficient trading decisions and portfolio manager’s lack of the freedom to react rapidly without a pre-notice.

What is notable for this research is that risk measures were only used for the adjustments to three-year returns. However, one regression model of the study implicates the findings of Fama and French (1992) that the cross-sectional variation of the mutual fund performance can be better explained with P/E, P/B and market capitalization rather than with beta. Though, the most important finding for this study is the inconsistency of the first and the last decile of the mutual fund sizes.
Limited diversification

The larger the mutual fund grows, the more complex its management and diversification gets. According to Pollet and Wilson (2008, 2968) portfolio managers have tendency to scale up their current holdings rather than coming up with new investment ideas when the size of the mutual fund grows. To some extent this is a good phenomenon, but this might end up troubles when the ownership of individual stocks raises too much. This phenomenon can be seen in slower adaptation of number of stocks as a response to the increase in assets under management.

The research was executed with different samples from the year 1975 to 2000. The main sources for the data was Center for Research in Security Prices (CRSP) and CDA database which is currently owned by Thomson Financial. All funds were investing in equities. The main measurement for the research was the fund size which was measured with total assets under management. (Pollet, Wilson 2008, 2945)

However, while the individual mutual funds tend to increase their current holdings as a response to the growth of assets under management, fund families tend to increase their number of funds rather than increasing the assets under management in their mature funds. This concerns especially larger and dominant fund families in the market. In addition, these new funds are usually diversified from the current fund offering which strengthens the diversification at the fund family level when the total assets under management of the fund family increases. Therefore, large fund families are security-wise diversified when their underlying mutual funds lack the diversification in different securities. The study explains this phenomenon with the diseconomies of scale related to the margin product of additional human capital in portfolio selection and investment ideas. (Pollet, Wilson 2008, 2968)

3.2 Performance

Risk shifting

Performance is the probably the most studied point of view of the mutual fund market and it is usually taken as one explanatory variable in researches where the actual focus concerns other
factors of mutual funds. One interesting point of view which is highly related to this particular study was presented by Huang, Sialm et al. (2011). The research shows that mutual fund industry has shifted its risk over time significantly between the years 1980 and 2009. In this matter, risk shifting refers to the exposure changes that are affecting the annualized volatility of the mutual funds. The research estimates that the risk shifting explains one third of the total annualized volatility of the examined sample of 2,979 U.S based equity funds.

The key finding of the research was that the risk shifting affects the performance of the mutual funds. The mutual funds that tend to shift their risk over time are prone to worse performance figures than the mutual funds having more stable risk profile. Poor performance is acquired particularly after the risk shift. In addition, more active mutual funds with higher active share are more likely to suffer from the risk shifting. This is speculated to be a consequence of poor trades driven by agency problem and tournament behavior in money flows rather than fund managers’ attempts to take advantages of occurring investment opportunities. (Huang et al. 2011, 2612)

From the investor point of view, the risk shifting over time cannot be seen absolutely harmful to the invested capital. There can be cases where the phenomenon is not causing performance consequences and the investor is able to form efficient portfolios accordingly with their risk tolerance and performance goals. However, risk shifting is something that the investor needs to be aware of because the shifting can interrupt the ability to assess the fund performance and the risk profile. (Huang et al. 2011, 2612)

Direct versus intermediate distribution

Within the financial industry, it is almost taken as granted that the average actively managed mutual fund in U.S equity market suffers from negative after-fee alphas. Still, a vast majority of investors is not behaving accordingly with this assumption. Guercio and Reuter (2014) challenged this assumption with their research where they compared the performance measures across directly sold and intermediated mutual funds.

The study includes 230 mutual funds from 1992 and 345 mutual funds from 2004. The data source was a combination of Financial Research Corporation and Center for Research in Security Prices. These funds were categorized into direct-sold or broker-sold segments based
on their assets under management on the unit class level. If the assets under management exceeded 75% in one of these segments, the categorization was done correspondingly. Data was also processed into mutual funds sold for retail investors and institutional investor after which the second group was excluded. (Guercio, Reuter 2014)

The key finding was that the mutual fund market has two retail investor segments based on their investment decisions. The first segment is driven by after-fee risk-adjusted performance results while the second segment rely their investment decisions to advisories and broker companies. According to the study, these segments have not been distinguished in earlier researches about underperformance of active fund management and it shows that the underperformance concerns only active mutual funds which are sold via brokerage. (Guercio, Reuter 2014)

One might ask, that how this phenomenon is possible? The study explains this by the increased likelihood of ignorance or irrationality in the broker-sold investor segment. It is obvious, that the losers of this equation are the investors and the winners are the advisors granted with higher commissions in actively managed mutual funds. Thus, the study identifies an agency problem between the brokers and the retail investors. However, this doesn’t mean that the clients would invest better without their financial advisories. (Guercio, Reuter 2014)

3.3 Management

Bank versus independently-managed mutual funds

Mutual funds are usually in competition about the same investors regardless of the management company structure. Therefore, there could be significant differences behind the mutual fund’s operation procedures, management company’s size and magnitude to the financial industry for example. One approach to the management companies was executed in a research by Melissa B. Frye (2001) where the focus was to evaluate whether bank-managed mutual funds are underperforming compared to other mutual fund companies.

The research was done with restricted sample out of 731 bank-managed and 3,368 nonbank-managed mutual funds. The restriction narrowed down the numbers to 97 bank and 699 nonbank-managed mutual funds. These funds included different types of asset classes and
investment approaches. Data was collected from Morningstar Inc. database from January 1991 to September 1999. (Frye 2001, 466)

The outcome of the research did not bring any evidence of bank-managed mutual funds’ underperformance. However, the study found that bank-managed mutual funds are conventionally more conservative in investment strategy and less likely to be exposed to risk measured in monthly standard deviation of returns. In addition, banks are more likely targeting retail investors rather than institutional investors and their usual clientele is attracted most likely by marketing information and bank’s reputation. (Frye 2001, 441)

*Management in team versus individually*

Consistently with the division between bank and nonbank-managed mutual funds we have studies about the portfolio management from the investment strategy management point of view. More specifically, Bliss et al. (2008) researched the performance characteristics between individually-managed and team-managed mutual fund portfolios. These characteristics concentrated to examine performance to provide new information for more profitable investment decisions but additionally it examined the risk profiles in terms of volatility and risk exposures.

This research was executed with the data fetched from Morningstar Inc. and Center for Research in Security Prices. The examined period took place between 1992 and 2003 where the number of actively managed mutual funds significantly increased from 1,168 equity funds to 2,639 equity funds. As an interesting curiosity in this period was, that the number of team-managed mutual funds increased more rapidly than individually-managed. The total compounded annual growth rate was 7,7% where team-managed mutual funds grew 13,8% and individually-managed mutual funds 3,3%. (Bliss et al. 2008, 112)

The research concluded that there was no evidence found that the management in group or individually would matter from the performance point of view. However, the risk profile was remarkably different in these two groups. Risk was measured by standard deviation of returns of the mutual funds. Team-managed mutual funds were exposed to less risk than their counterparties managed individually. Furthermore, team-managed mutual funds had lower cross-sectional differences in their performance figures and systematic risk loads. Additionally,
study found that the expenses and the front and back-end loads were lower in team-managed funds than individually-managed mutual funds. (Bliss et al. 2008, 111)

Summary

Based on these researches there can be done a couple of assumptions while selecting a mutual funds for own investments. Firstly, the mutual fund should not be too small nor too large, so it is not suffering from unwanted consequences of the unbalanced trade-off between margin value of trading and expenses. According to these researches, the safest choice would be in the medium sized mutual funds.

Secondly, the risk profile should be fitted by choosing of either a bank or nonbank managed mutual fund. This is mostly a matter of investor preferences and the goal of the investments. This same choice applies to the mutual funds with team or single portfolio manager organizations. However, it is beneficial to investigate whether the mutual fund has changed its investment strategy over time or has it kept the strategy consistent and untouched. This way the investor might reduce the possibility of risk shifting.

Finally, the mutual fund investor should avoid subscribing mutual funds via intermediates. Additionally, investor should be aware of the diversification policy of the mutual fund family, so that the mutual fund family applies sufficient diversification also on a mutual fund level rather than only mutual fund family level. The main characteristics can be found from the fund’s prospectus. By these selections the mutual fund investor would increase the possibility of a rational choice and better overall outcomes based on the previously introduced researches.
4 EMPIRICAL RESEARCH

Based on the theoretical framework and the previous research discoveries this study is intended to examine the efficient market hypothesis within Finnish equity fund market. According to the efficient market hypothesis, there should not be found any reoccurring and consistent anomalies. However, the mutual fund shell around the equity portfolio might affect the actual CAPM-based measures due to the factors related to mutual funds’ specifications. The main focus in this research concerns the factors related to relationship between the equity mutual fund’s size and the CAPM-based risk measurements. Additionally, other factors are also taken into account to support the interpreting of the research results. These factors are the expenses of the mutual fund and differing from the benchmark index which represents the activeness of the mutual fund.

The main focus in this study is to determine that are the mutual funds in the Finnish mutual fund market scaling perfectly or is the mutual fund shell around the portfolio biasing the market efficiency. Hence, the first research question is:

1. Is there any correlation between the size of equity mutual fund and the risk profile of the fund?

Based on the previous research there is a reason to expect that certain correlations can be found. Therefore, for further examination the next step is to determine the magnitude of these correlations. The second research question is:

2. What are the coefficients of the correlations?

Finally, these possible correlations with magnitudes are examined from the other point of views such as expenses and index tracking. This way the research ensures a solid base for explanation of the research findings. The second additional research question is:

1. Are there correlations between the size of equity mutual fund and the other measures of the fund?

With these research problems and their examined answers, it is possible to provide comprehensive conclusions about the researched framework. Additionally, there will be seen the next possible fields of further research.
4.1 Reference data

The reference data used in this research is exported from the Infront ASA. market data terminal. Infront ASA. is not calculating fund measures but they are acquiring these details from the Morningstar Inc. which is probably the most well-known investment fund database and broadly used within research matters as in the previously introduced ones. Therefore, all CAPM-based calculations for individual mutual funds are accomplished by Morningstar Inc against the benchmark indices of the mutual funds.

The export was accomplished on 17th of September 2017 which was Sunday which means that the values are from the last work day, Friday. This limits the examined period accordingly with the research frame to be the one-year range between 15th of September 2016 and 2017 in a way of a section sample. Due to the narrow time period, the sample is not expected to be highly survivor biased. All the CAPM-based calculations are done based on the latest NAVs of the funds which is 15th of September in the vast majority of the funds.

There are multiple data points in the export sample. The values are mostly on ratio and nominal scales of measurement. These data points have been used for limitations and the actual research calculations. The examined data points for calculations in this research are divided into explanatory variables and response variables. The actual data point names by Infront ASA. are presented in the captions. The variables are:

Explanatory variable
- Total assets under management (Total Assets)

Response variables
- Alpha of 12 months (Aplha12M)
- Beta of 12 months (Beta12M)
- Information ratio of 12 months (Info12M)
- Annualized Sharpe ratio (SharpeR)
- Standard deviation of 12 months (StdDev1Y)
- Total expense ratio (Total Expense)
- Tracking error of 12 months (TrackErr12M)
- Treynor ratio of 12 months (Treynor12M)
- Volatility of 30, 60 and 90 days (VOL30, VOL60 and VOL90)
There are multiple limitations made in the reference data. The base data is exported from the list in Infront ASA market data terminal which is named Finnish funds accordingly with its target market. This list gathers all the mutual funds which are available in sales in Finland. There are 1928 investment funds at the unit class level in total without limiting the list. These funds are directly sold by Finnish asset managers and mutual fund companies or directly issued by them but also by brokers who are distributing foreign mutual funds in Finland. Hence, there can be found Finnish issuing fund companies or other international issuers.

Second major limitation which has been made to the reference data is the limitation to equity mutual funds only. This is done by setting the categorization data point “fund type” of Morningstar Inc. to “equity”. This way all the other types are excluded from the data set including allocation, alternative, commodities, convertibles, fixed income, miscellaneous, money market and property.

In the reference data there are multiple mutual funds with different base currencies. Therefore, the currencies are converted to single base currency which is selected to be the most dominant base currency in the sample which is euro. The exchange rates used are the official euro foreign exchange reference spot rates issued by European Central Bank on 15th of September 2017 (European Central Bank 2018). The spot rates and the currency pairs used in the research can be seen in table 1.

<table>
<thead>
<tr>
<th>Pair</th>
<th>Spot rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR / AUD</td>
<td>1,4911</td>
</tr>
<tr>
<td>EUR / CHF</td>
<td>1,1480</td>
</tr>
<tr>
<td>EUR / GBP</td>
<td>0,88043</td>
</tr>
<tr>
<td>EUR / HKD</td>
<td>9,3506</td>
</tr>
<tr>
<td>EUR / JPY</td>
<td>132,92</td>
</tr>
<tr>
<td>EUR / NOK</td>
<td>9,4438</td>
</tr>
<tr>
<td>EUR / SEK</td>
<td>9,5058</td>
</tr>
<tr>
<td>EUR / USD</td>
<td>1,1963</td>
</tr>
</tbody>
</table>

*Table 1: Spot rates on 15th of September 2017*

In the reference data, there are also some funds that are either not providing all of their information to Morningstar Inc. for a reason or another or the funds are in too early stage that the calculations could be done for one-year period. Therefore, the sample size varies based on the data available of the mutual funds. As an example, Fidelity Investments is not providing expense details for Morningstar Inc. as mentioned in the theoretical background. Additionally,
there are exclusions where the details of the assets under management is not available about the investment funds for certain reason. However, these exceptions represent a minor share of the total sample. The descriptive statistics and the sample division into distinct variables of the total sample can be seen in the results section.

Some modifications needed to be accomplished in the reference data for preparing it to the actual examination. The largest change is that the total assets under management data points are converted into logarithmical scale to make the sample normally distributed. There are also some minor fixes done in the data related to the detail formats such as converting proportional amounts to corresponding decimal amounts and such in order to make the data more suitable for the statistical software used. All modifications were accomplished using Excel 2016.

In order to examine different thresholds of the mutual funds’ size in the sample, the full sample is divided into deciles based on the amount of the mutual funds. The division is similar kind than in the research of Indro, Jiang et al. (1999). Consequently, the research tests are executed both with a full sample and with the sample divided into deciles.

4.2 Hypothesis

According to the theoretical background and the efficient market hypothesis theory, the size of a mutual fund should not affect its features regarding to performance, risk and other measurements. These features should be distributed evenly across the whole sample. However, efficient market hypothesis theory cannot be applied directly to the mutual funds as a whole due to the administrative layers on top of the actual portfolios of common stock. These administrative layers are causing biases to the efficiency which were introduced as a part of behavioral finance faculty.

The previous research provides also guidelines for the assumptions of this research. According to Indro, Jiang et al. (1999) there is a reason to assume that the biasing effect would be emphasized at both ends of the sample. Also, there is a reason to believe that larger mutual funds are preferring portfolio management in teams rather than by individual portfolio managers which therefore lowers the risk profile in larger scale mutual funds accordingly with the management comparison by Bliss, Potter et al. (2008). This could be seen as a part of risk
management processes to distribute the portfolio management risks to follow a consensus of a group rather than individual portfolio managers.

Taking into account these attributes, the hypothesis is set to the first research question:

\( H_0: \text{No correlation can be found between risk measure and mutual fund size.} \)

\( H_1: \text{Correlation can be found between risk measure and mutual fund size.} \)

For the additional third research question the hypothesis is similar kind:

\( H_0: \text{No correlation can be found between other measure and mutual fund size.} \)

\( H_1: \text{Correlation can be found between other measure and mutual fund size.} \)

### 4.3 Methodology

The research is executed as a quantitative investigation of the introduced reference data. The research process follows the basic flow chart structure of the relationship investigation between two variables (Holopainen, Pulkkinen 2008, 248). According to the flow chart, there are six steps to follow:

1. Rationality analysis
2. Initial scatter plot analysis
3. Determining the suitable test
4. Execution of the selected test
5. Significance analysis
6. Interpretation and conclusion

The research would be terminated if one of the analysis would fail. For example, it is not reasonable to continue to the initial scatter plot analysis if the rationality analysis fails. Each of the steps are explained separately bellow:

1. **Rationality analysis**

The rationality of researching mutual fund size and each of the introduced measures as a variable pair can be seen from multiple dimensions such as on-going status of the mutual fund market, underlying theories about finance and behavior and previous research. These are introduced separately in the earlier stage of the research. Also, there cannot be seen any
restrictions why mutual fund’s size would not be able to affect the risk profile that it is exposed. The growth of the mutual fund forces it to change its processes and trading which can be seen as a factor that might cause consistent phenomenon across the mutual fund market. Hence, the rationality analysis can be seen as approved.

2. Initial scatter plot analysis

The initial scatter plot analysis was executed with Tableau Public 10.4 data visualization software. The analysis was accomplished for each of the response variables at a time. This gave initial results of the research outcome and strengthened the assumptions for creating hypothesis. Also, it exposed any inconsistencies in the reference data and helped to modify the research frame to more suitable way. In the scatter plot diagrams, there was visible possible correlations in some of the variable pairs using visualizing trend lines and confidence bands. Regardless that the possible correlations were visible only in a few variable pair, all variables were decided to take further to the testing phase to clarify the existence of the correlations.

In addition, Tableau Public 10.4 was used to get insights of the descriptive statistics of the data. It is fairly challenging to spot consistent patterns and phenomenon from a raw numerical data. That way computer software helped to compare and analyze different values of the research. The used methods were comparing values in different charts and tables, taking means and deviation numbers across the categories, limiting certain parameters one by one of the sample.

3. Determining the suitable test

All variables in the testing phase are measured in a ratio scale. Accordingly, for correlation examination between two variable the most suitable test would be Pearson correlation. Other bivariate correlation test options would have been Spearman’s and Kendall’s correlation test, but they are more suitable for variables limited to ordinal scale. Pearson correlation is usually referred as the most common correlation form (Holopainen, Pulkkinen 2008, 233). The Pearson’s correlation coefficient measures linear correlation between examined variables and it is measured between -1 and +1 representing the volume of the correlation. (Holopainen, Pulkkinen 2008, 234)

To identify the alterations between the different thresholds in size, Pearson’s correlation coefficient is measured in two streams: in the full sample and in the deciles of the sample. Therefore, the test is executed separately once for the full sample per variable pair and ten times
in the distinct deciles per variable pair. In total the testing phase consist of 121 Person’s correlation coefficient calculations.

More specific equation for Pearson correlation test can be found in appendix II.

4. Execution of the selected test

The execution of the testing phase is accomplished with IBM SPSS Statistics 23 which is the leading software of advanced statistics. In the software, there is a functionality for bivariate correlations including Pearson correlation. Tests are first accomplished with the full sample and then with the deciles of the sample. The deciles are divided by splitting the file into groups based on the decile data point added in the Excel 2016.

5. Significance analysis

The significance analysis is accomplished together with the correlation testing. The software functionality enables to include the tests of two-tailed significance in the correlation calculations. Thereafter, the significances are flagged with asterisks in the results. The flagged significances are segregated based on the confidence level into 0.01 level and 0.05 level.

6. Interpretation and conclusion

After the complete flow of the Pearson correlation test, the results are shown and interpreted in respect to the theoretical background and previous research. Thereafter, these results are explained and used for determining the hypothesis in each case and answering the research questions. Finally, all findings are concluded at the end. The conclusion proposes also themes for further research.
5 RESULTS

The research is accomplished with Pearson correlation tests. For the testing, the results section first introduces the descriptive statistics for an overview of the research sample’s key measurements and their features. After the descriptive statistics the actual results of the research are shown in a way of correlation matrixes. All the significant findings are explained, and other results are explained on a general level.

The results are divided into two categories: full sample and deciles. Thus, it is easier to see differences between distinct mutual fund size thresholds. The correlation across the full size shows the direction of the totality despite that the correlations in the subgroups might be correlating other way around. Furthermore, this setup provides more comprehensive results of the researched questions and reduces the possibility of coincidences occurred in this particular reference data.

5.1 Descriptive statistics

Full sample

The full sample is representing all of the equity based mutual funds which are available in sales in Finland and which are handing over their details for Morningstar Inc. Also, the sample size varies across the different measurements. The table 2 shows all the descriptive statistic details about these mutual funds.

The most interesting thing of the sample is the range in the mutual funds’ size. Some of the mutual funds are flagships of the Finnish mutual fund market and some of them are in their early days from inception. However, the median of the sample is significantly lower than the mean value, which means that the vast majority of the funds are placed at the smaller end of the sample.

The other interesting group of measure in terms of this researches point of view is the risk measures. The range of standard deviation of one year between the minimum and maximum shows that the mutual funds are varying in their risk policies. Some of the mutual funds are
Taking risk actively and some of the mutual funds are more conservative in risk tolerance wise. In general, the Finnish mutual fund market has obviously a tendency to stay below 10% of standard deviation where the mean and median are also placed. However, the shorter-term volatility of 30, 60 and 90 days are inclining slightly above the 10% in mean and median figures which describes that the mutual funds were more exposed to risk during the summer time after which the sample was taken. Other CAPM-based risk measures are distributed within the mutual funds. Beta of 12 months has central tendency figures that are relatively close to the market portfolio value, one. However, the range is broad. As a curiosity, the minimum value of beta is -0.05 which means that there is no equity based mutual funds in the sample which are following market portfolio inversely. Thus, it is difficult to be hedged against the systematic risk on average with equity based mutual funds in Finland. Though, market portfolio neutral options are available.

Also, other examined measures are offering insights of the sample. The expenses of the mutual funds are averaging at 1.63% which means that the expenses are at the level of actively managed mutual funds on average regardless that the equity-based index funds are not excluded from the sample. This can be also seen in tracking error of 12 months figures which is on the mean values at 4.25. Although, the existence of index linked mutual funds can be seen of the minimum value of the tracking error of 12 months. Hence, it can be assumed that the Finnish equity-based mutual fund market is oriented to active management rather than index following. On the other hand, this seems to be harmful for the industry due to the negative mean values of information ratio of 12 months, which represents that the activeness on portfolio management was not worth the risk on average.

Although, this sample shows also that the systematic risk of the examined period has been following the same line with the market portfolio on average which supports the model portfolio theory and the efficient market hypothesis. However, maximum values of Sharpe ratio and especially alpha of 12 months show that there were also highly profitable mutual funds that managed to outperform the market portfolio significantly within the financial year. Notably, there was also correspondingly unsuccessful mutual funds which should have invested in the market portfolio rather than follow their investment strategy in terms of performance and risk-reward ratios.
## Descriptive Statistics

<table>
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<tr>
<th></th>
<th>Alpha 12M</th>
<th>Beta 12M</th>
<th>Info 12M</th>
<th>SharpeR 1Y</th>
<th>StdDev 1Y</th>
<th>Total Assets EUR</th>
<th>Total Expense 12M</th>
<th>TrackErr 12M</th>
<th>Treynor 12M</th>
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a. Multiple modes exist. The smallest value is shown

### Table 2: Descriptive statistics

## Descriptive statistics by Total Assets EUR

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<td>1.09%</td>
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<td>5.00%</td>
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</table>

a. Multiple modes exist. The smallest value is shown

### Table 3: Descriptive statistics by Total Assets EUR
Deciles

The categorization into deciles shows the details of different thresholds in terms of descriptive statistics and central tendency values. The basic characteristics of the deciles can be seen in table 3 and the other examined variables can be seen in appendix I. Hence, it is possible to compare deciles with each other or against the full sample.

Since the sample has been divided into deciles based on the amount of mutual funds, relatively large portion of the assets under management are weighted in the larger end of the deciles. The largest decile covers almost 80% of the total assets under management while the first four deciles are measured in fractions of percent. This way the funds in the opposite ends of the scale are having totally different operating environments in terms of size. Between the end deciles there are the mean and median of the total assets under management which can be seen from the table 2. Due to the pressure to the higher end in terms of assets under management, the mean is located in the ninth decile and the median is located in the end of the fifth decile.

The table of other examined measures in appendix I shows the corresponding values for each measure per decile. There are few picks to mention. Firstly, the riskiest mutual fund decile appears to be the last decile measured in mean value of one-year standard deviation. This can be seen also in the short-term volatilities of 30 and 90 days where the tenth decile appears to be the riskiest in mean values. However, in terms of mean values of Sharpe ratio, the risk taking is not rewarding the tenth decile with higher returns and it loses in the risk-reward ratio to the ninth decile which is exposed to significantly less risk than the tenth decile. This same phenomenon can be seen with median values. This might be an implication caused due to the diseconomies of scale introduced by Perold and Salomon (1991).

Another curiosity in the table concerns the standard deviations of the deciles. It seems like mutual funds in the larger deciles are less likely deviating from each other than in the lower deciles concerning Sharpe ratios. This same phenomenon seems to concern also standard deviations of alpha of 12 months, total expense ratio and tracking error of 12 months among the mutual funds within same size decile. This could mean that there are less mutual fund type selection and investment strategies among the larger equity based mutual funds.
On the other hand, there cannot be done too extensive assumption based on the mean and deviation values of the deciles. However, these are such trends that would be beneficial ideas for further research. However, in this research, these trends are not examined further.

5.2 Correlations

As the first thing in the correlation analysis, it is notable that the mutual fund sizes are not normally distributed. Though, other examined measures are relatively normally distributed. For Pearson correlation test, it is crucial to have variables which are normally distributed to get reliable result. Therefore, the size scale of the sample is log-transformed to make the variables normally distributed in the sample.

In order to get the best understanding about the log-linear relations between the measured variables, the Pearson correlation test is accomplished with the full sample and in deciles of the full sample. The full sample focuses to reveal any phenomenon across the total scale of the examined mutual fund sample. The correlations in deciles show whether the appearances are concentrated between specific mutual fund size thresholds or that they are occurring consistently across the full scale in every decile.

**Full sample**

The Pearson correlation test across the full sample reveals some interesting features of the Finnish equity-based mutual fund market. These results are shown in table 4. The findings show that there are two statistically significant findings in the sample.

The first one concerns the total expense ratio of the mutual funds. It seems like the correlation is significant at the 0,01 level with two tailed significance test and the direction is negative at 0,095 coefficient. This means that the total assets under management increase on a logarithmic scale leads to a decrease in total expense ratio. Although, the coefficient is not strong but notable.
Correlations

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</table>

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

Table 4: Correlations

Another statistically significant discovery concerns tracking error of 12 months. The correlation test shows that this correlation is also negative, and its coefficient is 0.071 at the significance level of 0.05 with two tailed significance test. In other words, when the total assets under management increase on a logarithmic scale, the tracking error of 12 months decreases. Likewise, with expense ratio, the coefficient is relatively weak. For interpreting the result, it is essential to take into account the direction of tracking error. The lower tracking error of the mutual fund, the higher the similarity in returns between the mutual fund and the benchmark index. Thus, the correlation direction states that the larger the mutual fund, the more it tracks the benchmark index.

The test does not show other significant results at the 0.05 level or lower with two tailed significance test. However, the results concerning beta of 12 months is significant at the level
of 0,056 which is notable in this research. Although, it cannot be generalized outside of the perimeters of this research. The correlation is positive and its coefficient is 0,06, so it is also relatively weak. However, it means that when the total assets under management increase on a logarithmic scale, the beta of 12 months of the mutual fund is increased.

All other variable pair results are rejected and no other log-linear correlations can be found. The margin of error increases unsustainably with the other tests which cannot be accepted. Only way to find more results within these variable pairs would be by changing the parameters of the research framework.

*Deciles*

The decile approach to the Pearson correlation with the sample leads to 110 distinct correlation tests. There are multiple statistically significant results in the correlation matrix. This correlation matrix can be seen in table 5 with coefficient values, significances and sample sizes.

In the correlation matrix, all the accepted correlations for rejecting $H_0$ are significant at the 0,05 level with two tailed significance test. From all the correlations, the most important finding concerns standard deviation of 12 months. In the largest decile, there can be seen a negative correlation where the coefficient is 0,193 which can be interpreted relatively high within the sample. This can be seen as a clear implication of diseconomies of scale accordingly with the findings of Perold and Salomon (1991) where the phenomenon concentrates to the larger end of the sample. Additionally, this can be seen in contrary to the results of the full sample regarding other risk measure, beta of 12 months.

Another interesting finding in the correlation matrix relates to the total expense ratio measures. The correlation tests show that in the third decile there can be seen a significant correlation coefficient between the total expenses and the log-transformed total assets under management. Furthermore, the correlation is positive and the coefficient is relatively strong among the sample at 0,239. This is another finding in contrary to the previous results with the full sample where the correlation appeared negative across the full sample. Hence, there could be a factor in the third size category what distinguishes the direction of the correlation from the full sample.
Correlations by decile

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<td>.070</td>
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<td>-.040</td>
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<td>.478</td>
<td>.095</td>
<td>.688</td>
<td>.048</td>
<td>.767</td>
<td>.099</td>
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<td>.234</td>
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<td>.365</td>
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<td>105</td>
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**. Correlation is significant at the 0.01 level (2-tailed).
* . Correlation is significant at the 0.05 level (2-tailed).

Table 5: Correlations by decile

There are also findings in the divided sample regarding measures which are not correlating in the full sample. First one can be located in the ninth decile of the sample concerning Treynor ratio of 12 months. The Pearson correlation test shows that the ninth decile has a positive correlation and relatively strong coefficient among the sample between assets under management and Treynor ratio of 12 months. This means that within the range of ninth decile, an increase in assets under management leads to an increase in Treynor ratio of 12 months. Thus, the result is aligned with the results concerning beta of 12 months which has a weak and positive correlation across the full sample due to the fact that beta and Treynor ratio are linked together equation-wise.

Finally, there can be found statistically significant correlation regarding volatility of 60 and 90 days in sixth decile. Both of these correlations are positive and coefficients can be seen as
notable. The coefficient for volatility of 60 days is 0.196 and for 90 days it is slightly stronger, 0.234. These results are not in contrary with other results in the full sample, because it supports the inclining trend within risk and mutual fund size in the smaller and mid-size deciles.

In case the significance margin is increased a bit for rejecting the $H_0$, there can be found a couple of correlations which can be useful for explaining the findings and the phenomenon. These correlations cannot be generalized outside the perimeters of this research. Nevertheless, they can polish more significant results and their directions.

There is a correlation in the last decile regarding Treynor ratio of 12 months. The correlation is positive and the coefficient is 0.191 exceeding the 0.05 level of two tailed significance test only by 0.1%. This could implicate that the tenth decile is not completely suffering from its size.

In the ninth decile, there is also something notable. At the significance level of 0.066 there can be seen a negative correlation coefficient of 0.180 in beta of 12 months. This could implicate that the inclining risk trend starts to change the direction already within the ninth decile. Additionally, also in ninth decile there is a positive correlation coefficient of 0.175 in alpha of 12 months. The significance of the correlation is at the level of 0.074 which has exceeded the level of 0.05 error margin only by 2.5%-units. This supports the findings from the point of view, where the diseconomies of scale are not completely affecting the ninth decile yet.

Finally, there can be mentioned to be a local inconsistency in the fourth decile regarding volatility of 90 days. The correlation test shows that there would be a negative correlation coefficient of 0.176 with the significance level of 0.072. This discovery seems to be not aligned with other smaller deciles.

Likewise, with the full sample, other results are rejected due to the increasing level of significance margin. However, these results are prone to react when the parameters of the research framework are changed. Therefore, this setup still leaves some stones unturned.

### 5.3 Interpretation of the results

The results cannot be described as univocal. However, according to this research sample we can straight away reject the assumption of strong market efficiency concerning Finnish equity-based mutual fund market. The sample shows multiple correlations of different kinds across the
sample, which implicates that the mutual funds are not scaling perfectly in log-linear response to the growth measured in assets under management. All the results are summarized in the table 6.

### Result summary of coefficients

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<td>Deciles</td>
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<td>Full sample</td>
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<td><strong>Treynor12M</strong></td>
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<td><strong>VOL60</strong></td>
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<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Significance levels (2-tailed) | <0.01 | <0.05 | <0.08

*Table 6: Result summary of coefficients*

In respect to the significance level of 0.05 there are certain correlations related to mutual fund’s risk bearing in Finnish market. The most obvious evidence in the sample shows that the total risk of the largest mutual fund group has a tendency to decrease when the mutual fund’s size increases. Also, there is an evidence that the Treynor ratio increases in the ninth decile in response to mutual fund’s growth. There is also a reason to believe that the trend remains within tenth decile. This might be in accordance with the diseconomies of scale. However, no implications can be done within this research that the size would be harmful to mutual funds regarding risk-reward ratios and performance. Besides, Treynor ratio shows that nevertheless risk measured in standard deviation decreases the beta-risk-adjusted returns are increased. Though, the situation might be changed when the index-based mutual funds are excluded from the sample. Another explanation might be that the larger mutual funds have more likely grouped
portfolio managers, so the likelihood of team management grows along with the mutual fund’s size. This theory would be supported by the findings of Bliss et al. (2008).

In general, there are no evidence that the smaller end would show inconsistencies in terms of risk profile. There can be done only a weak assumption that the mutual fund’s growth increases the tendency to be exposed to the systematic risk or to be locked in with the benchmark index. This assumption is structured based on the results of tracking error, volatility and beta. Also, descriptive deviation numbers of the deciles support this theory. This leads to a thought that the smaller end of the mutual funds is more diversified. Those funds might include more specialized investment strategies and unique point of views for niche investor segment.

Though, based on the variables measured uniqueness comes with a price. The smaller end of the mutual funds is more likely more expensive than the larger end of the mutual funds. This can be seen in the correlation of expense ratio. This makes sense due the fact that economies of scale apply at least to some extent also within mutual fund industry with administration, trading on information and other fixed and changing cost for example. This finding is aligned with the theoretical framework introduced by Bodie et al. (2012, 91). Despite, the third decile seems to be an exception without any explanation. It might be that the mutual fund shell causes some reasons related to trading, administration or other arrangements to increase the pricing.

In order to get better understanding of the results, the weaknesses of the Pearson correlation test must be digested. Pearson correlation shows only the direction of the bivariate correlation and the coefficient but not the actual ratio between the variables. Therefore, even though it can be seen that the last decile reveals negative correlation between the assets under management and the standard deviation of the returns, it does not reveal the slope of the relation. When the size of the mutual fund is increased by one euro, it cannot be said that by how much the risk decreases in the last decile.

Additionally, Pearson correlation shows only linear correlation between two variables. Therefore, it can be misleading if the correlation is actually non-linear or polynomial. For the best results, each statistical modelling should be accomplished separately before making too extensive assumptions.

Furthermore, these correlation tests cannot be applied when comparing two individual mutual funds from the Finnish market. They are only providing guidelines for the likelihoods of measured risk and other factors. It is notable, that the coefficients are weak and indicative while the highest coefficient reaches almost one quarter compared against full correlation of 100%.
Additionally, the error margin of the Pearson correlation test must be taken into account before making too comprehensive generalizations. The significance levels are differentiated in the table 6 to highlight the probability of an error to reject the H\(_0\) with wrong justifications for distinct coefficients.

However, in respect to the results, the research can be seen as successful in terms of providing answers to the research questions. They were also partially aligned with the previous research, but also partially in contrary which could be a result of limitations or other perimeters of the research framework. In addition, the research leaves some questions unanswered for example relating to total expense ratios in the third decile of the sample. Though, the most important thing in terms of these results is that there is no evidence that these correlations could be harmful or severe for individual or institutional investors.
6 CONCLUSION

This research shows that the equity-based mutual fund market in Finland is not perfectly obeying the efficient market hypothesis. There were found evidence that the size affects consistently to the features of the mutual fund related to the risk profile and risk-adjusted performance measures. This could be explained by behavioral finance theory and some of its branches. Also, the regulative environment for equity-based mutual funds in Finland and European Union in general are heavily binding in terms of risk management and allocation limits. These limits and rulesets may affect to the market-based relations of the mutual fund industry. However, it cannot be proofed that whether the reason behind the phenomenon is driven by administration, portfolio management or unitholders of the mutual fund.

The research shows also that annual expenses were declining with the growth of the mutual funds, which is a consistent phenomenon across the fund industry, nevertheless not against efficient market hypothesis theory. This phenomenon might be driven by the circumstance that the expenses are not growing with the same intensity than assets under management in the whole of the mutual fund shell. The economies of scale are apparently applying to some extent.

The clearest phenomenon in terms of risk profile is that the mutual funds have a tendency to be more locked in with the benchmark indices while the mutual funds grow. This could be driving the largest mutual funds to appear reducing total risk in returns’ deviation within the largest decile of the industry. However, based on the descriptive sample analysis, the Finnish mutual fund market seems to be more orientated to the active portfolio management rather than passive index tracking. Furthermore, there appears to be more deviation in terms of selection in the smaller end of the size scale. Based on these factors, it can be concluded that size adds the likelihood to be more passive management oriented and reduces the selection related to different strategies and risk profiles.

There is also weaker evidence about increasing risk along with the mutual fund size. It seems like the volatility and beta figures are positively correlating towards the larger end of the scale. However, this cannot be generalized outside of this research because of the broader error margin and unclarity of the phenomenon.

However, this research could not replicate the phenomenon from the previous research related to the diseconomies of scale. Total risk declines in the largest end, but the risk-adjusted returns cannot be concluded to be declining. In addition, this research could not find any minimum
threshold to reach before gaining competitive risk-adjusted returns. Though, in order to identify certain threshold, there should be more distinct research setups and size groups rather than dividing the sample simply into deciles. This applies also to the general perspective of the research meaning that the results are extremely sensitive to the research framework and parameters.

Hence, the research leaves open multiple possible research opportunities for further investigation. Firstly, the examined sample consist all the index-linked mutual funds which are in some cases excluded from the sample. Therefore, it would be reasonable to separate these funds from the sample and investigate the risk profiles in exclusively in actively managed and passively managed mutual funds. This could also clarify some of the findings of this research.

Secondly, this research does not take into account distinct fund strategies or focused stocks. It would be one point of view more to investigate the relationship between risk profiles and size from the mutual fund strategy perspective. This would have possibilities to distinguish phenomenon between large and small-cap focusing mutual funds for example. Furthermore, there is such like possibilities to change parameters and discover new findings even though the focus market and the sample would be the same.

One approach would be to examine the risk measures and the correlations in time series. This way it would be easier to identify phenomenon related to risk shifting, tournament behavior and other moment sensitive trends. In this research it is difficult to say whether these phenomena are affecting the examined mutual funds or not.

Finally, this research leaves open the discovery about the contrary correlation of the third decile regarding to expense ratios. Based on the error margin and the volume of the coefficient the discovery can be described as significant. What differentiates the third decile from the general trend of declining expenses as a response to the increasing mutual fund size? It would be interesting to find out, which is the exact range of the correlation and what are the reasons of it?
REFERENCES

Literature


Legislation


Web page


### APPENDIX I: DESCRIPTIVE STATISTICS BY DECILE

**Descriptive statistics by decile**

<table>
<thead>
<tr>
<th>Decile</th>
<th>Alpha 12M</th>
<th>Beta 12M</th>
<th>Info 12M</th>
<th>Sharpe R</th>
<th>StdDev 1Y</th>
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<th>TrackErr 12M</th>
<th>Treynor 12M</th>
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a. Multiple modes exist. The smallest value is shown
APPENDIX II: PEARSON CORRELATION TEST EQUATION

According to Graham Upton and Ian Cook (2014) the equation of Pearson correlation coefficient (r) is calculated as follows:

\[ r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} \]

, where

\[ S_{xy} = \sum_{j=1}^{n} x_j y_j - \frac{1}{n} \left( \sum_{j=1}^{n} x_j \right) \left( \sum_{j=1}^{n} y_j \right) \]

, and

\[ S_{xx} = \sum_{j=1}^{n} x_j^2 - \frac{1}{n} \left( \sum_{j=1}^{n} x_j \right)^2 \]

Finally, \( S_{yy} \) is defined correspondingly with \( S_{xx} \).