NEO-KEYNESIAN MACROECONOMICS OF INTER-NATIONAL FACTOR ALLOCATION

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Abstract

In the Neo-Keynesian macroeconomic model demand shocks have no long-run real effects on production, real wages and real interest rates. Therefore, there should be no effects on factor migration either. During the short-term adjustment, however, the temporary real effects make effects on factor migration also possible. Under Adaptive Expectations, migration may act as an inter-national alternative to intra-national adaptation of price expectations in the market adjustment process. In this case, the factors are induced to move to opposite directions in order to adjust the economy to the initial shock. A positive demand shock lowers perceived real wages thus inducing emigration of labour, and makes perceived real interest rates rise thus inducing immigration of capital. The final amounts of migration depend on the relative shares of intra- and inter-national adjustment.

Key words: AEH, migration, short-term adjustment
JEL classification: F41, R23
1 Introduction

Factor migration and spatial allocation of economic resources are vital elements of the global economy. A leading intuition of economic theory is that, under free migration, the market mechanism supports a stable and efficient spatial equilibrium where no benefits can be derived from relocation. Any exogenously originated inter-national differences in market circumstances are always re-equalized in the market.

In standard textbook presentations, factor migration is most often studied in partial equilibrium models. These models are comparative static analyses of the equilibrating forces between locally described markets. Another line in the literature is general equilibrium models, such as the one-sector model, which explain migration by differences in factor endowments, and the two-sector model that analyses migration effects of demand shocks with special emphasis on the price mechanism (McCann, 2001, pp. 208-221).

This paper examines inter-national factor allocation in a Neo-Keynesian macroeconomic framework. The model deals with the simultaneous market adjustment in the goods and factor markets. The virtue of the model is twofold: first, it provides an accurate description of the working of the price system, and second, it facilitates a comprehensive analysis of labour and capital migration.

The paper proceeds as follows. The Neo-Keynesian framework is constructed in Section 2. For a benchmark, the classical regime of long-term adjustment is presented in Section 3. The short-term adjustment path is then studied in close scrutiny in Section 4, including intra-national adjustment in the original spirit of the Adaptive Expectations Hypothesis, and inter-national adjustment facilitated by factor migration. Section 5 concludes the findings.
2 The model

The model is an overlapping generations elaboration of the seminal Neo-Keynesian macroeconomic framework (Samuelson, 1958; Diamond, 1965; Felderer & Homburg, 1987; Heijdra & van der Ploeg, 2002; Laurila, 2004). The model describes the interplay of firms and consumers in factor and goods markets. The factor market includes both labour and capital markets.

**Firms** maximize profits

(1) \[ \pi = pq - wL - rK \]

applying the production function

(2) \[ q = f(L,K). \]

In (1) and (2), \( p \) is current price and \( w \) and \( r \) are the nominal market yields for labour \( L \) and capital \( K \), respectively. Concerning (2), the standard assumptions are made, namely constant returns to scale and \( f_1, f_2 > 0, f_{11}, f_{22} < 0, f_{12} = f_{21} > 0 \).

**Consumers** maximize a two-period utility function

(3) \[ U = u(c_1, 1-l) + \frac{1}{1+\rho} v(c_2) \]

subject to the budget constraints
In (3) and (4), consumption is $c_1$ in period 1 and $c_2$ in period 2, and $\rho$ denotes time preferences. Total available time in period 1 is fixed to 1, and $l$ is leisure time. Consumption price is $p_1$ in period 1 and $p_2$ in period 2. In period 1, consumers save portion $s$ of their labour income, which they consume with interest in period 2. The first order optimum conditions are $u_2/u_1 = w/p_1$ for time allocation and $u_1/v' = p_2(1+\rho)/p_1(1+r)$ for saving.

In the labour market, labour demand is given by maximization of (1) under (2), which straightforwardly yields the demand function

\begin{equation}
(5) \quad w = pf, \\
\end{equation}

written in terms of nominal wages. By (5), firms hire labour according to its marginal revenue product, which declines when $L$ increases. Thus, the demand curve slopes downwards.

Labour supply originates from (3) and (4), and reads $l = l(w; p_1, p_2, r)$. Take $r$ fixed and denote the price vector $(p_1, p_2)$ by $p^\ast$, expected price. This reflects the difference in the time horizons of people’s and firms’ decision-making - people must screen price information over two periods, whereas firms need only current data. In aggregate terms, $L = L(w/p^\ast)$, which says that labour supply in the market depends on expected real wage. Inversely, $w/p^\ast = g(L)$, or

\begin{equation}
(6) \quad w = p^\ast g(L),
\end{equation}
where the right-hand-side tells the expected market value of the opportunity cost of labour time. Assuming $g' > 0$, the labour supply curve is upward sloping in $L-w$ space.

In the capital market, capital demand is again straightforwardly given by (1) and (2), which yield $r = pf_2$ saying that the capital demand curve has a negative slope in $K-r$ space. Solving for $p$ gives

\begin{equation}
(7) \quad p = r/f_2,
\end{equation}

which turns the curve upward sloping in $K-p$ space. Capital supply arises from the saving decisions of the previous period. From (3), individual’s saving demand function is $s = s(r,p^e,w)$. Fixing $w$ and writing in terms of aggregate capital supply yields $K = K(r/p^e)$, which says that present capital supply, provided by the currently old generation depends on the real interest rate they anticipated in the previous period. In inverse form, $r/p^e = \phi(K)$, which says that the market yield on capital must compensate the opportunity cost of supplying it, and the assumption $\phi' > 0$ makes the curve slope upwards in $K-r$ space. Solving for expected prices gives

\begin{equation}
(8) \quad p^e = r/\phi(K),
\end{equation}

which says that the capital supply curve slopes downwards in $K-p$ space.

**Aggregate supply** in the goods market is determined by the production function (2) and by the factor market equations (5)-(8) thus determining the aggregate supply (AS) curve of the economy. Totally differentiating equations (5) and (6) and solving for proportionate changes yields

\begin{equation}
(9) \quad \frac{dL}{L} = \frac{\mu^s}{\mu^d + \mu^s} \left[ \mu^d \left( \frac{dp}{p} - \frac{dp^e}{p^e} \right) + \frac{dK}{K} \right]
\end{equation}
for the labour market equilibrium. In (9), $\mu^D = f_1/Lf_{11}$ denotes the demand elasticity and $\mu^S = g/Lg'$ denotes the supply elasticity in the labour market. Totally differentiating equations (7) and (8) gives

$$dK = \frac{\eta^S}{\eta^D + \eta^S} \left[ \eta^D \left( \frac{dp}{p} - \frac{dp^e}{p^e} \right) + \frac{dL}{L} \right]$$

for the capital market equilibrium. In (10), $\eta^D = f_2/Kf_{22}$ denotes the demand elasticity and $\eta^S = \varphi/K\varphi'$ denotes the supply elasticity in the capital market.

Totally differentiate the production function (2), manipulate and write

$$\frac{dq}{q} = \beta \frac{dL}{L} + (1 - \beta) \frac{dK}{K},$$

where $\beta = wL/q$ is the share of labour income and $1 - \beta = rK/q$ is the share of capital income of total factor income. Substitution of the equilibrium equations (9) and (10) to (11) gives

$$\frac{dq}{q} = \beta \mu^D \mu^S (\eta^D + \eta^S) + (1 - \beta) \eta^D \eta^S (\mu^D + \mu^S) \left( \frac{dp}{p} - \frac{dp^e}{p^e} \right) + \beta \frac{\mu^S}{\mu^D + \mu^S} \frac{dK}{K} + (1 - \beta) \frac{\eta^S}{\eta^D + \eta^S} \frac{dL}{L}$$

for the expression of aggregate supply. The first term on the right-hand-side of (AS) implies that the aggregate supply curve is vertical unless the price changes are anticipated asymmetrically by the demanders and the suppliers of the factors. The AS curve is upward sloping, if people’s price expectations lag behind the true current prices. Changes in factor supply affect production with
positive relationship. In the case that capital and labour shift in opposite directions, the effects depend on the relative shares of labour and capital income, and on the demand and supply elasticities in the factor markets.

**Aggregate demand** is determined by the aggregate conditions of the real and monetary sides of the economy. The equilibrium condition of the real side, the expression of the IS curve, reads

\[
q = c(q^D) + i(r) + b,
\]

where \( c \) is aggregate consumption based on disposable income \( q^D = Lw/p^e + Kr/p^e \), which says that current disposable income consists of labour income of the currently young generation and capital income of the currently old generation. By assumption, \( c' > 0 \). Investment is denoted by \( i \) with the assumption that \( i' < 0 \). Parameter \( b \) denotes any exogenous demand shocks. Initially, \( b = 0 \). Trade is ignored and the budget constraint of the local public sector is not specified. Totally differentiate (12) and get

\[
dq = \frac{q}{p^e} c \left[ \beta \left( \frac{dL}{L} + \frac{dw}{w} - \frac{dp^e}{p^e} \right) + (1 - \beta) \left( \frac{dK}{K} + \frac{dr}{r} - \frac{dp^e}{p^e} \right) \right] + i'dr + db.
\]

By the second term on the right-hand-side of (13), the IS curve is declining in \( q-r \) space. The last term says that positive consumption shocks shift the curve in outwards and vice versa. The first, bracketed term on the right-hand-side tells that the curve also shifts, if disposable income changes because of changes in factor employment and/or in real factor rewards.

The equilibrium condition of the monetary side, the LM curve, reads
(14) \( m/p = l(q,r) \),

where the supply of real money equals the demand of liquid money, reasoned by transaction use depending on production \( q \) and inter-temporal use depending on the interest rate \( r \). The assumptions \( l_1 > 0 \) and \( l_2 < 0 \) are applied. Totally differentiating equation (14) yields

\[
(15) \quad dr = \frac{1}{l_2} \left( \frac{dm}{m} - \frac{dp}{p} \right) - \frac{l_1}{l_2} dq,
\]

which indicates that the LM curve is upward sloping in \( q-p \) space, and that increases in real money shift the curve outwards and vice versa. Substituting (15) to (13) gives

\[
(AD) \quad \frac{dq}{q} = \frac{l_2}{l_2 + \nu \nu_1} \left[ \frac{c'}{p'} \left( \beta \left( \frac{dL}{L} + \frac{dw}{w} \right) - \frac{dp^*}{p^*} \right) + (1 - \beta) \left( \frac{dK}{K} + \frac{dr}{r} - \frac{dp^*}{p^*} \right) + \frac{1}{q} \left( \frac{i m}{p} \left( \frac{dm}{m} - \frac{dp}{p} \right) + db \right) \right]
\]

for aggregate demand in the goods market. The AD curve has a negative slope in \( q-p \) space. Positive demand effects caused either by monetary \((dm)\) or real \((db)\) shocks shift the AD curve outwards and vice versa. The first two terms in the braces on the right-hand-side of (AD) imply that the curve also shifts, it changes in disposable income are anticipated. If the changes in labour and capital income are of opposite signs, the net effect depends on their relative shares.
3 Classical long-term effects of a demand shock

The classical regime of the present framework gives a benchmark for the consideration of the price adjustment mechanism. For the sake of simplicity, it is assumed that the national economy in question is of atomistic size so that the factor rewards outside are taken as given. The factor rewards are assumed non-transportable. Recall that there is no trade of goods either. It is also assumed that $p=p^e$ in equations (6) and (8) and $dp/p=dp^e/p^e$ in (AS).\(^1\)

Figure 1 presents the market adjustment. The Figure consists of two sets of quadrants. Panel (i) includes the labour market described by equations (5) and (6), the production function described by (2) and the goods market described by expressions (AS) and (AD), and the real wage $\omega=w/p=f_1$, listed clockwise from northeast. Panel (ii) includes the capital market given by equations (7) and (8), the production function, the IS-LM framework given by equations (12)-(13) and (14)-(15), and the real interest rate $\rho=r/p=f_2$, listed clockwise from northeast.

(Figure 1 here)

In Figure 1, the initial inter-national equilibrium is given by the market price set $(p_0, w_0, r_0)$. Thus, the real wage is $\omega^*$ and the real interest rate is $\rho^*$ everywhere. Employment is $L_0$ and the capital stock is $K_0$ implying that production is $q_0$. Suppose that there emerges an exogenous demand shock so that $db>0$ in (13) and thus in (AD). The effect is shown by the outward shift of the IS curve from $IS_0$ to $IS_1$ in Panel (ii) and the consequent shift of the AD curve from $AD_0$ to $AD_1$ in Panel (i) in Figure 1.

\(^1\) Actually, this is the Perfect Foresight Hypothesis, but the more modern and less stringent Rational Expectations Hypothesis would be equivalent in terms of graphics.
The increase of demand in the goods market causes the prices to rise from $p_0$ to $p_1$. In the classic regime of the model this causes labour demand to shift outwards from $D^L_0$ to $D^L_1$ and labour supply to shift backwards from $S^L_0$ to $S^L_1$ in Panel (i) of Figure 1. Nominal wages rise from $w_0$ to $w_1$, and the labour market equilibrium shifts vertically from $e_0$ to $e_1$.

In Panel (ii) of Figure 1, the shift of the IS curve from $IS_0$ to $IS_1$ is accompanied by the inward shift of the LM curve from $LM_0$ to $LM_1$ because of the rise in the price level from $p_0$ to $p_1$. The nominal interest rate rises from $r_0$ to $r_1$ so that the real interest rate remains unchanged at $\rho^*$. Therefore, the demand for capital shifts backwards from $D^K_0$ to $D^K_1$, and the supply of capital shifts outwards from $S^K_0$ to $S^K_1$. The capital market equilibrium shifts vertically from $\hat{e}_0$ to $\hat{e}_1$.

In Figure 1, real wages, employment and production remain unchanged at $\omega^*$, $L_0$ and $q_0$, respectively. Unchanged remain also real interest rates, capital use and production at $\rho^*$, $K_0$ and $q_0$, respectively. The conclusion is that the demand shock has no real effects in the economy. In particular, there are no effects on factor migration either.
4 Neo-Keynesian short-term adjustment

4.1 Intra-national adjustment

Consider now the Neo-Keynesian short-term adjustment path towards the above classical long-term equilibrium. In order to concentrate on the price adjustment mechanism, apply the Adaptive Expectations Hypothesis\(^2\) to describe the intra-national adjustment path. In Figure 2 below, the short-term adjustment is illustrated by assuming that factor demand and factor supply respond asymmetrically to the exogenous price shock. By (5) and (7), the firm sector is assumed to be perfectly foresighted, while (6) and (7) say that the household sector is assumed to lack such rationality. This is to say that \(p = p^e\) does not hold during the adjustment.

(Figure 2 here)

Start again from the positive demand shock implying that the IS curve should shift from \(IS_0\) to \(IS_1\) in Panel (ii) and the AD curve should shift from \(AD_0\) to \(AD_1\) in Panel (i) of Figure 2. In Panel (i), the perfectly foresighted firms anticipate that the price level rises according to the long-term aggregate supply curve \(AS^{LT}\) from \(p_0\) to \(p_1\). Thus, the demand for labour shifts from \(D^L_0\) to \(D^L_1\). Likewise, in Panel (ii), since the firms correctly anticipate that the nominal interest rate rises from \(r_0\) to \(r_1\) due to the decrease of real money that makes the LM curve shift from \(LM_0\) to \(LM_1\) according to the long-term \(LM^{LT}\) curve, the demand for capital shifts backwards from \(D^K_0\) to \(D^K_1\).

In the first place, the shock surprises the people as factor suppliers so that they undervalue the price effects. In Panel (i) of Figure 2, as people anticipate only the direction but not the full amount of the price effect, they respond by reducing labour supply so that the labour supply curve shifts from \(S^L_0\) to \(S^L'\). On the other hand, in Panel (ii), the supply of capital does not react at all, because it is fixed.

\(^2\) The Hypothesis is undeniably out-dated, but it serves well the purposes of this paper.
by the saving decisions made in the previous period. Note that the variable on the vertical axis of Panel (ii) is \( p \), while (8) is written in terms of \( p^e \) during the adjustment.

The momentary equilibrium in the labour market shifts from \( e_0 \) to \( e' \) Panel (i) and that in the capital market shifts from \( e_0 \) to \( e' \) on Panel (ii) of Figure 2. At the anticipated prices \( p' \), employment increases from \( L_0 \) to \( L' \); the nominal wage rises to \( w' \). The real wage becomes \( \omega' \), which is lower than \( \omega^* \). Respectively, in the capital market of Panel (ii), capital use reduces from \( K_0 \) to \( K' \), the nominal interest rate is \( r' \) and the anticipated real interest rate is \( \rho' \), which is higher than \( \rho^* \). The production function shifts inwards from \( f(L,K_0) \) to \( f(L',K) \) in Panel (i) and outwards from \( f(L_0,K) \) to \( f(L',K) \) in Panel (ii) of the Figure.

Recalling (AS) and assuming that the labour market is more elastic than the capital market, the net increase of production factors shifts the AS curve from \( AS_0 \) to \( AS' \) in Panel (i) of Figure 2. Furthermore, recalling (AD), the temporary increase in disposable income makes the AD curve shift to \( AD' \), which is beyond \( AD_1 \). Thus, it is the \( AD' \) curve that actually sets prices to \( p' \) along \( AS' \). This also makes the LM curve shift from \( LM_0 \) to \( LM' \) in Panel (ii) thus yielding the nominal interest rate \( r' \). As a result, production has temporarily increased from \( q_0 \) to \( q' \) in both Panels.

Adaptive Expectations Hypothesis says that people gradually correct their actions until the long-term equilibrium is reached. In the labour market people respond to the decreased real wage by further reducing their labour supply. The momentary equilibrium \( e' \) thus starts to move along \( D^L_1 \) towards \( e_1 \) in Panel (i) of Figure 2. Likewise, in the capital market in Panel (ii), the young cohort responds to the anticipated rise in real interest rates by altering its saving patterns thus increasing their supply of capital so that the momentary equilibrium moves from \( e' \) towards \( e_1 \) along \( D^K_1 \). It must be noted that the adjustment path in the capital market may take longer than that in the labour market, because saving is inter-temporal in nature whereas labour supply can be altered currently.
As a result of the gradual error-correcting mechanism, factor use moves back towards its initial stage, that is employment falls from $L'$ towards $L_0$ and the capital stock increases from $K'$ towards $K_0$. Since employment and capital stock change, the production function rotates from $f(L,K')$ towards $f(L,K_0)$ and from $f(L',K)$ towards $f(L_0,K)$. Likewise, the graph of the real wage rotates from $\omega'$ towards $\omega^*$ and that of the real interest rate rotates from $\rho'$ towards $\rho^*$. The classical long-term equilibrium with no real effects is thus eventually reached after the adjustment path has been gone through.

Note that the temporary factor market responses are of opposite signs - labour employment tends to increase while capital use tends to decrease. Recalling equations (AS) and (AD), it is thus possible that production is temporarily reduced, not increased as in Figure 2. Figure 3 presents this kind of an adjustment path.

(Figure 3 here)

Start again from the momentary equilibrium given by $e'$ and $e'$ in Figure 3. Assuming now that the capital market is more elastic than the labour market, there occur a relatively small increase of employment in the labour market and a big reduction of capital use in the capital market. Therefore, the AS curve shifts backwards to $AS'$, which is beyond $AS_t$. Taken that this reflects also in the disposable consumption incomes, the AD curve (alas the IS curve) shifts only to $AD'$, which is below $AD_t$. In this case, production is reduced from $q_0$ to $q'$. Nevertheless, the result is again a temporary reduction in the real wage and a temporary increase in the real interest rate.

The gradual error-correcting mechanism makes employment eventually fall from $L'$ towards $L_0$ and the capital stock increase from $K'$ towards $K_0$. Therefore, the production function rotates from
$f(L,K')$ towards $f(L,K_0)$ in Panel (i), and from $f(L',K)$ towards $f(L_0,K)$ in Panel (ii) of Figure 3. Likewise, the graph of the real wage rotates from $\omega'$ towards $\omega^*$ and that of the real interest rate rotates from $\rho'$ towards $\rho^*$. The classical long-term equilibrium with no real effects is thus again reached along the adjustment path.

4.2 Inter-national adjustment

Consider now the alternative of inter-national market adjustment path served by the possibility of free factor mobility. Return to the momentary equilibrium described by points $e'$ and $\dot{e}'$ in Panels (i) an (ii) of Figure 3 (likewise in Figure 2, too). In the labour market, at point $e'$ in Panel (i), people see that the local real wage $\omega'$ has fallen below $\omega^*$. Since the real wage in the rest of the economy remains at $\omega^*$ and supposing that people are fully aware of that, motives for emigration arise. Observing at $p_1$, which is the price level that makes local labour demand and labour supply commensurable, the induced amount of emigration is $L_1 - L_0$, measured in terms of labour time units.

In the capital market in Panel (ii) of Figure 3, the short-term equilibrium at $e'$ says that the warranted local real interest rate has improved from that in the rest of the economy, $\rho' > \rho^*$. Observing the situation from the side of the firms, and reading at the price $p$ that is relevant to them, the demand for capital exceeds local supply, which is based on over-estimated real interest rate. Taken that capital is fully mobile, the obvious solution is to hire capital from the rest of the economy. The consequent immigration of capital amounts to $K_0 - K_1$.

The induced emigration of labour $L_1 - L_0$ and immigration of capital $K_0 - K_1$ immediately reproduce the long-term classical equilibrium, where employment is $L_0$, capital use is $K_0$ and production is $q_0$. The real wages and real interest rates are again equalised everywhere to $\omega^*$ and $\rho^*$, respectively. In
In this case, in spite of the fact that production remains unchanged, there still are real effects because of the movement of the factors: labour has emigrated and capital has immigrated.

The above result is due to the asymmetric effects of the demand shock on the factor markets. A positive demand shock, the outward shift of the AD curve, increases prices while nominal wages lag behind thus causing a momentary fall in the local real wages anticipated by the labour suppliers. Given that higher real wages are available elsewhere, emigration of labour is induced. On the other hand, the shock, the outward shift of the IS curve, makes the nominal interest rate rise while prices lag behind thus causing a momentary rise in the capital suppliers’ anticipated local real interest rate, which induces less than adequate local supply and thus immigration of capital. Emigration and immigration act as inter-national vent for surplus and deficit, respectively.
5 Conclusions

The analysis of inter-national factor allocation in the Neo-Keynesian macroeconomic model makes several contributions. First, in this model, the price adjustment mechanism is very explicit in the general equilibrium context. The adjustment is most straightforward in the classical regime of the model, where exogenous demand shocks have no long-term real effects on production, real wages, real interest rates and factor migration. The result of long-term neutrality is reasoned by the Perfect Foresight Hypothesis or the Rational Expectations Hypothesis.

Second, the long-term neutrality result still holds in the Neo-Keynesian regime, where the original idea of the Adaptive Expectations Hypothesis is applied as a description of the intra-national error-correction adjustment path towards the long-term equilibrium. The short-term effects on production, real wages and real interest rates are gradually mitigated by the intra-nationally operating error-correcting mechanism of the factor markets. There are necessarily no effects on migration.

Third, the original spirit of intra-national adjustment under AEH can be substituted by the international explanation given by factor migration caused by the temporarily perceived differences in real factor rewards. If the adjustment happens inter-nationally, there will be permanent emigration and immigration effects, while the effects on production, real wages and real interest rates remain temporary. In this case, a positive demand shock induces permanent immigration of capital and emigration of labour and vice versa.

Fourth, both the intra- and inter-national adjustment mechanisms may naturally take place at the same time thus replacing each other. The final shares of the two mechanisms in the total adjustment process and thus the amount of factor migration induced depend on the relative flexibilities of the two mechanisms. It can be argued that the intra-national mechanism is not very flexible because
updating labour contracts and life-cycle saving plans is tedious and time-consuming, while international mobility can be practiced somewhat more rapidly. Therefore, migration should take a substantial role, at least in the capital market. Of course, this remains an empirical question.
References:


Figure 1: Classical macroeconomic effects of a demand shock

Panel (i)

Panel (ii)
Figure 2: Short-term effects of a demand shock

Panel (i)

Panel (ii)
Figure 3: Another case of short-term effects

Panel (i)

Panel (ii)