Charitable Conservatism, Poverty Radicalism and Good Old Fashioned Inequality Aversion

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Abstract

In his Presidential Address to the European Economic Association, Tony Atkinson introduced the idea of a “Charitable Conservatism” position in public policy, which “exhibits a degree of concern for the poor, but this is the limit of the redistributional concern and there is indifference with respect to transfers above the poverty line.” This contrasts with the perspective of poverty indices, which give zero weight to those above the poverty line, which we call “Poverty Radicalism,” and with standard “Inequality Aversion” where the weights decline smoothly as we move up the income scale. The object of this paper is, first, to clarify the interrelationships between Charitable Conservatism, Poverty Radicalism and Inequality Aversion. We do this by showing how the patterns of welfare weights to which each of these gives rise are related to each other. Secondly, we are concerned to demonstrate the implications of these different views for optimal income taxation. In terms of levels and patterns of marginal tax rates, we show that Charitable Conservatism and Poverty Radicalism are on a continuum, and by choice of low or high Inequality Aversion one can approximate either outcome fairly well.

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1 We are grateful to Katharina Nordblom for her very useful comments on the earlier version of this paper.
1. Introduction

The Bergson-Samuelson social welfare function, expressing social welfare as a function of individual utilities, has been the workhorse of welfare economics since its introduction. Its special parametrisations have proved particularly fruitful in the analysis of optimal taxation. For example, much use has been made of functional forms where the social marginal utility of income falls smoothly as income rises. Indeed choice of this pattern of welfare weights, capturing "inequality aversion", may be described as the currently accepted practice. Of course, the rate at which welfare weights fall, the magnitude of inequality aversion, is still a matter of choice. Several parametrisations exist which permit convenient representation of the degree of inequality aversion, for example, the "constant elasticity" class of functions, and these have been used extensively.

The "smoothly falling welfare weights" class of social welfare functions have thus long dominated the analysis of optimal taxation. However, in recent years at least two alternatives have been suggested as capturing better certain classes of value judgements. The first of these is to be found in the growing literature on poverty indices. Starting with Sen (1976), the literature consciously gives a zero social marginal utility to incomes above a critical level ("the poverty line"), thus allowing a focus on incomes below this level. Sen (1976) codified this as the "focus axiom", and it is formalised in terms of welfare weights by Atkinson (1987). This alternative has not been without its critics. Stern (1987), in a defence of standard parametrizations, expresses dissatisfaction with poverty indices because welfare weights fall to zero (and may do so discontinuously for some indices) at the poverty line. It is argued that this is too extreme; it can be avoided by using standard parametrizations and letting the degree of inequality aversion increase, which gives greater and greater weight to the poor, while ensuring that (i) weights fall smoothly and (ii) they do not fall to zero at a finite value of income.

Forcing welfare weights to fall to zero well before the highest incomes are reached may indeed be considered extreme relative to the standard inequality aversion. We will refer to it as "Poverty Radicalism"; some among us find it appealing. But another alternative to the standard view has been discussed by Atkinson (1990) in his Presidential Address to the European Economic Association - this time an alternative not in the Radical but in the Conservative direction. He reasons as follows.

"It may be that complete distributional indifference characterises the social welfare function of some Conservative governments, but there is a more charitable position, which believes
that the government should be concerned with poverty but not with redistribution. This charitable conservative position exhibits a degree of concern for the poor, but this is the limit of the redistributitional concern and there is indifference with respect to transfers between those above the poverty line".

Thus, in Atkinson's (1990) characterisation of "Charitable Conservatism", the welfare weights are constant at a high level for all incomes below the poverty line, they then fall (discontinuously) to a low level at the poverty line, whence forth they are constant at this low level.

The weights pattern described above is of course equally open to Stern's (1987) criticisms levelled at poverty indices. The weights characterising Charitable Conservatism fall discontinuously at the poverty line and are constant thereafter. At the same time, the only difference between Poverty Radicalism and Charitable Conservatism seems to be the magnitude of the weight given to above the poverty line incomes - the pattern of the weights (constancy) is the same for both.

It has to be said that the conventional inequality aversion view has a certain advantages. It avoids the discomfort of discontinuous changes in weights. It has a pleasing unity and flexibility as captured by the inequality aversion parameter. Can the Poverty Radicalism and Charitable Conservatism views not be accommodated simply by increasing or decreasing the degree of inequality aversion, without having to resort to such severe departures? And what difference does it make which view one holds? In particular, does it make a huge difference where it counts - in the pattern of optimal taxation to which each system of weights gives rise? It is the object of this paper to attempt an answer to these questions.

The plan of this paper is as follows. Section 2 presents a formalisation and discussion of the three types of welfare weight patterns. Section 3 sets up the basic optimal income taxation model and then moves to a presentation of numerical results. Section 4 concludes.

2. Alternative Patterns of Welfare Weights

Let the social valuation of an individual's wage be denoted $\psi(n)$, and let social welfare $^2$ be

$$S = \int_{0}^{\infty} \psi(n) f(n) dn$$

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$^2$ If social welfare is strictly a function of individual utility, then $\psi'(n)$ depends on $u(n)$. This dependence complicates the analysis. We return to this in the context of numerical simulations. Now (1) may be described as 'non-welfarist'. In fact this is implicit in a number of approaches to measuring inequality.
where \( f(n) \) is density of \( n \) and the integration is over the range of \( n \). It is natural to suppose that \( \psi \) is increasing in \( n \), and a representation of egalitarianism is to make \( \psi \) concave in \( n \). For differentiable functions we can say equivalently that \( \psi'(n) \) is decreasing in \( n \). With this notation, one definition of a welfare weight for unit change in income is simply:

\[
\varphi(n) = \frac{\psi'(n)}{E\psi'(n)}
\]

where \( E \) denotes the expectation operator. The interpretation is that a unit increment in \( n \) leads to an increase in social valuation of \( \psi'(n) \). But to get the value of this in terms of income requires normalisation. One natural procedure is to normalise by the average of \( \psi'(n) \) in the population. Put another way, \( \varphi(n) \) measures the social value of giving a unit of income to an individual with income \( n \), relative to the social value of dividing it equally among all individuals.

Not surprisingly, the behaviour of the function \( \varphi(n) \) is an important determinant of the pattern of optimal taxation. Focussing on this function will also allow us to clarify the differences between Charitable Conservatism, Poverty Radicalism, and Good Old Fashioned Inequality Aversion. Let us start with a conventional representation of \( \psi(n) \) as a "constant inequality aversion" function:

\[
\psi(n) = \frac{n^{1-\beta}}{1-\beta}, \beta \neq 1
\]

\[
= \ln(n), \beta = 1
\]

where \( \beta \) is the measure of inequality aversion. Then

\[
\varphi(n) = \frac{n^{-\beta}}{E(n^{-\beta})}.
\]

For example, when \( n \sim \Lambda(\mu, \sigma^2) \), i.e. a lognormal distribution with parameters \( \mu \) and \( \sigma^2 \), then

\[
\varphi(n) = \exp[-(\mu + 0.5\sigma^2)]n^{-\beta}
\]

This pattern of "smoothly declining" welfare weights is shown in Figure 1. Clearly the
weights are above 1 when \( n \) is low, and below 1 when \( n \) is high, the switch over point occurring when \( n \) reaches the \((-\beta)\)th order geometric mean of \( f(n) \). This point is denoted \( n_s \).

Figure 1 Good old fashioned inequality aversion

Suppose now that there is a critical value of income, \( n_{pov} \), known as the "poverty line". The reason why this value of income is critical depends on the behaviour of the valuation function, and therefore the welfare weights. We suppose that \( n_{pov} \) is less than \( n_s \). Atkinson's (1990) representation of Charitable Conservatism is shown in Figure 2. As can be seen, the weights jump to a lower value at \( n_{pov} \). Formally, we can write

\[
(1 - N_{pov}) \varphi_c + N_{pov} \frac{\varphi}{k} = 1
\]

where \( N_{pov} \) is a fraction of people who are below the poverty line and \( k \) is an indicator of the degree of concern for those below the poverty line.
Finally, consider the implicit weights in the Foster, Greer and Thorbecke (1984) class of poverty indices, which require:

\begin{equation}
\psi(n) = -\left(\frac{n_{\text{pov}} - n}{n_{\text{pov}}}\right)^\alpha ; n \leq n_{\text{pov}}
\end{equation}

\[ = 0 ; n > n_{\text{pov}} \]

The weights are thus:

\begin{equation}
\phi(n) = \frac{\{\alpha[(n_{\text{pov}} - n)/n_{\text{pov}}]^\alpha - 1 / n_{\text{pov}}\}}{E\psi'(n)} ; n \leq n_{\text{pov}}
\end{equation}

\[ = 0 ; n > n_{\text{pov}} \]

These weights are plotted for \( \alpha = 1 \) in Figure 3. It will be seen that the weight pattern is the same as the Charitable Conservative case. The only difference is that the poverty index gives a weight of zero to those above the poverty line and a rather higher weight to those below.

One feature of both sets of weights, in Figures 2 and 3, is the discontinuity at \( n_{\text{pov}} \). Stern (1987) criticises this in the context of poverty indices, but the criticism is equally applicable to Atkinson's Charitable Conservatism representation; and it has to be said that widely
different weights for incomes which are infinitesimally apart are difficult to justify. In fact, in the case of the poverty index in (8) this can be remedied by choosing $\alpha > 1$. For example, when $\alpha = 2$ we get a pattern as shown in Figure 4. The weighting function is now continuous, although it may be objected that it is non-differentiable at $n_{pov}$. This is also taken care of when $\alpha > 2$.

Figure 3 Poverty radicalism

Figure 4 Modified Poverty radicalism
But what of the Charitable Conservative position? There seems to be no way of avoiding discontinuity within the terms of Atkinson's definition. But suppose in fact that we were to modify this definition. Suppose that we imposed only, the requirement that after \( n_{pov} \) the weights should remain constant at their value at \( n_{pov} \). In order to ensure that the weights sum to one their values below \( n_{pov} \) will have to be somewhat higher. Modify the poverty valuation function (7) as follows:

\[
\psi(n) = kn - \left( \frac{n_{pov} - n}{n_{pov}} \right)^{\alpha}; n \leq n_{pov}
\]

\[
= kn; n > n_{pov}
\]

Thus

\[
\phi(n) = k + \frac{\alpha\left[\left(\frac{n_{pov} - n}{n_{pov}}\right)^{\alpha-1} / n_{pov}\right]}{E\psi'(n)}; n \leq n_{pov}
\]

\[
= k; n > n_{pov}
\]

\[
\phi(n) = \frac{\psi'(n)}{E\psi'(n)}; n \leq n_{pov}
\]

\[
= \frac{k}{E\psi'(n)}; n > n_{pov}
\]

This weight pattern is shown in Figure 5 for \( \alpha = 2 \). Comparing Figure 5 with Figure 4 shows how different values of \( k \) can convert Charitable Conservatism to Poverty Radicalism! In fact, when \( \alpha = 1 \) the weight pattern in (10) generates the discontinuous shape of Figure 2 for \( k > 0 \), and Figure 3 for \( k = 0 \), showing again that the parameter \( k \) is key in distinguishing between Conservatism and Radicalism. If it is required that the weights be differentiable, a value \( \alpha > 2 \) will suffice.
The above discussion of welfare weights serves to anchor Charitable Conservatism, Poverty Radicalism and Good Old Fashioned Inequality Aversion in a common framework. But how different are these three in terms of their implications for optimal income taxation policy?

3. Implications for Optimal Income Taxation

3.1 Basic model

We follow the Mirrlees (1971) model of optimal income taxation. There are a continuum of taxpayers, each having the same preference ordering, which is represented by a utility function \( u = U(x) - V(y) \) defined over consumption \( x \) and hours worked \( y \), with \( U_s > 0 \) and \( V_y < 0 \) (subscripts indicating partial derivatives). Individuals differ only in the pre-tax wage \( n \) they can earn. There is a distribution of \( n \) on the interval \((0, \infty)\) represented by the density function \( f(n) \). Gross income is \( z = ny \).

Each \( n \) individual maximises utility by choice of hours worked, solving

\[
\max_{x,y} u = U(x) - V(y) \quad \text{subject to} \quad x = ny - T(ny),
\]

where \( T(ny) \) is the tax schedule.
Suppose that the aim of policy can be expressed as maximizing the following social welfare criterion

\[
S = \int_0^\infty \psi(u(n)) f(n) dn
\]

where \( \psi(.) \) is an increasing and concave function of utility. The government cannot observe individuals’ productivities and thus is restricted to setting taxes and transfers as a function only of earnings, \( T(z(n)) \). The government maximizes \( S \) subject to the revenue constraint

\[
\int_0^\infty T(z(n)) f(n) dn = R
\]

where in the Mirrlees tradition \( R \) is interpreted as the required revenue for essential public goods. The more non-tax revenue a government receives from external sources, the lower is \( R \). In addition to the revenue constraint, the government faces incentive compatibility constraints. These in turn state that each \( n \) individual maximizes utility by choice of hour.

Totally differentiating utility with respect to \( n \), and making use of workers’ utility maximization condition, we obtain the incentive compatibility constraints,

\[
\frac{du}{dn} = -\frac{yV_y}{n} = g(u, y).
\]

Since \( T = ny - x \), we can think of government as choosing schedules \( x(n) \) and \( y(n) \). In fact it is easier to think of it choosing a pair of functions, \( u(n) \) and \( y(n) \), which maximize welfare index (13) subject to the revenue requirement (14), the incentive compatibility condition (15) Introducing Lagrange multipliers \( \lambda \), \( \alpha(n) \) for the constraints (14) and (15) and integrating by parts, the Lagrangean becomes

\[
L = \int_0^\infty [W(u) + \lambda(ny - x)) f(n) - \alpha' u - \alpha g] dn + \alpha(x)u(x) - \alpha(0)u(0)
\]

Differentiating with respect to \( u \) and \( y \) gives the first-order conditions\(^4\)

\(^3\) The 1.order condition of individual’s optimisation problem is only a necessary condition for the individual's choice to be optimal, but we assume here that it is sufficient as well. Assumptions that assure sufficiency are provided by Mirrlees (1976). Note also that while we here presume an internal solution for \( y \), (6) remains valid even if individuals were bunched at \( y=0 \) since, for them, \( du/dn=0 \).

\(^4\) \( x = h(u, y) \)
\[ L_u = [W' - h_y \lambda]f(n) - \alpha'(n) = 0 \]
\[ L_y = \lambda(n - h_y)f(n) + \alpha(n)(V_y + yV_{yy}) = 0 \]

Integrating in (17)

\[ \alpha(n) = \int_{-\infty}^{\infty} \left[ \psi' - \frac{\lambda}{u_x} \right] f(p) dp \]

This latter satisfies the transversality conditions $\alpha(o) = \alpha(\infty) = 0$

From the first order conditions of government’s maximization, we obtain the following condition for the optimal marginal tax rate $t(z) = T'(z)$; [Note: \( \frac{t}{1-t} = \frac{1}{1-t} - 1 = \frac{U_x n}{V_y} - 1 \)]

\[ \frac{t}{1-t} = (1 + \frac{yV_{yy}}{V_y}) \frac{U_x}{\lambda n f(n)} \int_{-\infty}^{\infty} \left[ \frac{\lambda}{U_x} - \psi' \right] f(p) dp \]

Multiplying and dividing (20) by $(1 - F(n))$ we can to write the formula for marginal rates;

\[ \frac{t}{1-t} = \left[ 1 + \frac{E^u}{E^c} \right] \left[ 1 - F(n) \right] \frac{U_x}{\lambda n f(n)} \int_{-\infty}^{\infty} \left[ 1 - \frac{\psi' U_x^{1(p)}}{\lambda} \left( \frac{1}{U_x^{1(p)}} \right) f(p) dp \right] \]

where $E^u$ is the uncompensated supply of labour and $E^c$ in turn is the compensated elasticity. It should be clear from (21) that the variation of the optimal marginal tax rate with the level of income is a complex matter. Applying (21) it appears that there are four elements on the right hand side of (21) that determine optimum tax rates: elasticity and income effects (A&C), the shape of the skill distribution (B&C) and social marginal weights (C). The C-term in (21) tends to favour rising marginal rates. This is so especially when income is low or moderate.
We may omit income effects in the labour supply by considering quasi-linear preferences in consumption. This special case is extensively used in both theoretical and applied optimal tax literature. Then it is possible to deduce on the basis of (21) that with Pareto distributions marginal tax rates rise with income at high levels of income. Once we allow income effects the analysis is more complicated. The optimal marginal tax rates in more general cases, as in (21), become considerably more difficult to interpret because labour supply can vary with skill and because of income effects. The quasi-linear assumption is also restrictive because it eliminates declining marginal utility of consumption (utility is linear in $x$), which is a key motivation for redistribution.

It is clear that explicit solutions to the optimal income tax problem are difficult to obtain without simplifying assumptions. The terms in (21) simplify if we assume quasi-linear preferences with constant the elasticity of labour, $U_x = 1$, the marginal tax rate formula is

$$ (22) \frac{t}{1-t} = \left[\frac{1}{E^c} \left( 1 - \int_{x_n}^c \frac{[1 - F(n)]}{nf(n)} \frac{[1 - \varphi]f(p)dp}{(1 - F(n))} \right) \right] $$

In the case of the unbounded Pareto distribution, $f(n) = \frac{1}{n^{1+\alpha}}$ for $\alpha > 0$, $\frac{1 - F(n)}{nf(n)} = \frac{1}{\alpha}$ is constant. Hence, the optimal marginal tax rate depends only $C$. In the classical utilitarian case $\varphi$ is constant for all $n$, then the marginal tax rates are uniformly zero.

3.2 Some Special Cases

If we assume the Rawlsian social objective\(^5\) then the factor $C_n$ is constant and when $U_x = 1$. 

\(^5\) Maximizing utility of worst off person in the society is not the original version of Rawls (1972). It is a kind of welfarist version of Rawls. “To interpret the difference principle as the principle of maximin utility (the principle to maximize the well-being of the least advantaged person) is a serious misunderstanding from a philosophical standpoint.” Rawls,1982)
then the pattern of marginal tax rates depends only on $B$, that is, on the shape of the $n$-distribution. The distribution of $n$ plays a major role in determining the optimal tax

\begin{equation}
\frac{t}{1-t} = \left[ 1 + \frac{1}{E_x^c} \right] \left[ 1 - \frac{F(n)}{nf(n)} \right]
\end{equation}

We specify further the case with a maximin criterion so that the upper part of the $n$-distribution is the unbounded Pareto distribution and the utility function is $u = x - y^{1-\frac{1}{\varepsilon}}$. ($E^c = \varepsilon$)

Then

\begin{equation}
\frac{t}{1-t} = \left[ 1 + \frac{1}{\varepsilon} \right] \frac{1}{a}
\end{equation}

Hence using the Rawlsian social welfare function we do not obtain the rising part of the U-shaped marginal tax rates as in Diamond (1998). As expected, the optimal top marginal tax rate is increasing in $\phi$ and decreasing in $\varepsilon$. It also depends negatively on $a$, which is a measure of the thinness of the tail of the $n$-distribution.

If we take a charitable conservative position we know from (6) that the last term in (23) is equal to $(1 - \phi_x)$. It can be calculated when we know $k$ and $N_{pov}$. The optimal tax formula becomes

\begin{equation}
\frac{t}{1-t} = \left[ 1 + \frac{1}{\varepsilon} \right] \frac{(1-\phi_x)}{a}
\end{equation}

Table 1 illustrates with some parameter values top marginal rates in Rawlsian and charitable conservative cases.

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6 In the general additive case with maximin $C$ is $\int (f(p)/u_x)dp$. It is declining with $n$ since $u(x)$ is concave and the integral term declines in $n$. This might suggest declining marginal rates. See also Boadway-Jacquet, 2008.
Table 1  Rawlsian and charitable conservative top marginal tax rates

<table>
<thead>
<tr>
<th>Pareto parameter</th>
<th>Elasticity $\varepsilon$</th>
<th>marginal tax rates % Rawls</th>
<th>marginal tax rates % Charitable conservative (k=1/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=3</td>
<td>0.25</td>
<td>62.5</td>
<td>45.4</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>50</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>40</td>
<td>25</td>
</tr>
</tbody>
</table>

It is of interest to note that if the Pareto distribution applies over the whole range of $n$, the optimal marginal tax rate is increasing up to the poverty line. The importance of the result underlines the fact that the fit of Pareto distribution over the whole range of income turns out to be quite poor.

The results in Table 1 depend on the chosen distribution of wages. Next we replace a Pareto distribution by the Champernowne distribution. As commonly known the lognormal distribution fits reasonable well over a large part of income range but diverges markedly at the both tails. The Pareto distribution in turn fits well at the upper tail. Champernowne (1952) proposes a model in which individual incomes were assumed to follow a random walk in the logarithmic scale. Here we use the two parameter version of the Champernowne distribution. This distribution approaches asymptotically a form of the Pareto distribution for large values of wages but it also has an interior maximum. As the lognormal, the Champernowne distribution exhibits the following features: asymmetry, a left humpback and long right-hand tail. But it has a thicker upper tail than in the lognormal case.

The probability density function of the Champernowne distribution is given by

$$ f(n) = \theta \left( \frac{\mu^\theta n^{\theta-1}}{(\mu^\theta + n^\theta)^2} \right) $$

in which $\theta$ is a shape parameter and $\mu$ is a scale parameter. The cumulative distribution
function is

\[
F(n) = 1 - \frac{\mu^\theta}{(\mu^\theta + n^\theta)}.
\]

For the distribution ratio

\[
\lim_{n \to \infty} \frac{1 - F(n)}{n f(n)} = \lim_{n \to \infty} \frac{\mu^\theta + n^\theta}{\theta n^\theta} \to \frac{1}{\theta}.
\]

Eq (28) shows that the Champernowne distribution approaches asymptotically a form of Pareto distribution for large values of wages. Calculating the ratio \(\frac{1 - F(n)}{n f(n)}\) with different parameter values of \(\mu\) and \(\theta\) we obtain the marginal tax rates with different social objectives.

**Table 2 Optimal marginal tax rates with social objectives (Rawlsian, Poverty Radicalism, Charitable Conservatism) and the Champernowne distribution \((N_{pov}=0.15, \epsilon=0.3)\)**

<table>
<thead>
<tr>
<th>F(n)</th>
<th>Rawlsian</th>
<th>Rawlsian</th>
<th>k=0.01</th>
<th>k=0.01</th>
<th>k=0.25</th>
<th>k=0.25</th>
<th>k=0.5</th>
<th>k=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>94.7</td>
<td>93.3</td>
<td>94.4</td>
<td>93</td>
<td>84.8</td>
<td>81.4</td>
<td>69.9</td>
<td>64.7</td>
</tr>
<tr>
<td>0.20</td>
<td>91.5</td>
<td>85.9</td>
<td>91</td>
<td>85.1</td>
<td>76.9</td>
<td>65.4</td>
<td>58.3</td>
<td>44.2</td>
</tr>
<tr>
<td>0.50</td>
<td>81.2</td>
<td>74.1</td>
<td>80.2</td>
<td>72.8</td>
<td>57.2</td>
<td>47</td>
<td>35.9</td>
<td>27.1</td>
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<tr>
<td>0.75</td>
<td>79.8</td>
<td>66.1</td>
<td>73.2</td>
<td>64.7</td>
<td>47</td>
<td>37.7</td>
<td>27.1</td>
<td>20.2</td>
</tr>
<tr>
<td>0.90</td>
<td>70.4</td>
<td>61.4</td>
<td>69.1</td>
<td>60</td>
<td>42.5</td>
<td>33</td>
<td>23.6</td>
<td>17.1</td>
</tr>
<tr>
<td>0.95</td>
<td>69.2</td>
<td>60</td>
<td>67.9</td>
<td>58.4</td>
<td>41.1</td>
<td>31.8</td>
<td>22.6</td>
<td>16.3</td>
</tr>
<tr>
<td>0.99</td>
<td>67</td>
<td>58.2</td>
<td>65.6</td>
<td>56.6</td>
<td>38.7</td>
<td>30.1</td>
<td>21</td>
<td>15.3</td>
</tr>
<tr>
<td>0.999</td>
<td>58.7</td>
<td>52.2</td>
<td>57.1</td>
<td>50.5</td>
<td>30.7</td>
<td>25.6</td>
<td>15.6</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Note that the marginal tax rates in different cases are far from zero for the top 0.1 per cent. These results help to show how misleading is the result that the marginal tax rate should be zero at the top when there is an upper bound to possible incomes.
3.3 Numerical Simulations

In more general settings, as is often the case in non-linear optimal taxation analysis, we have to rely on numerical simulations and we now turn to these. The calculations were carried out for the case

\[ u = -\frac{1}{x} - \frac{1}{1-y} \]

where the elasticity of substitution between consumption and leisure is 0.5. The variable \( n \) is taken to be distributed according to a lognormal distribution, with the mean of the logarithm of \( n \) being -1 and its standard deviation 0.4. The assumption about government revenue as a share of national income (\( g \)) is about 10 percent. It is assumed to be spent on public goods in a way that does not affect the rest of the model.

In the previous section a very special case for Charitable Conservatism was chosen so that the private social marginal utility of income was constant. With the utility function (29) it depends on consumption and the relation with \( n \) is not too obvious. In other words now \( \psi'(n) \) depends on \( u(n) \).

Let us consider first of all the level of the marginal tax rate schedule. It is seen in Figure 6 that Inequality Aversion with \( \beta = 2 \) is a half-way house between modified Charitable Conservatism and Poverty Radicalism at least in the specifications that we have used. In fact, other calculations (not reported here), show that increasing \( \beta \) by small amounts above 2 does not change tax rates by much, but increasing \( \beta \) to infinity i.e. approaching the maximin solution, does bring the resulting optimal schedule closer to that of Poverty Radicalism. Notice also that the pattern of marginal tax rates is fairly similar - they decline as income increases - no matter which pattern of welfare weights is used. The guaranteed income \( x(n_o) \) is higher in Inequality Aversion and modified Poverty Radicalism cases than in modified Charitable Conservatism case.
Figure 6 Marginal tax rates

**Modified Charitable Conservatism** \((x(n_0) = 0.08, \ g = 0.1, \ \sigma = 0.4)\) [with a chosen poverty cut off at \(n = n_{pov}\) so that \(F(n_{pov}) = 0.38\) and a pattern of weights as in Figure 5]

**Inequality Aversion** \((\beta = 2)\) \((x(n_0) = .13, \ g = .1, \ \sigma = .4)\)

**Modified Poverty Radicalism** \((x(n_0) = 0.127, \ g = 0.1, \ \sigma = 0.4)\) [with a chosen poverty cut off at \(n = n_{pov}\) so that \(F(n_{pov}) = 0.27\) as depicted in Figure 4 and equation (9) with \(k = 0\)]

Figures 7 and 8 display marginal tax rates for Charitable conservative position in case with \(k=1/3\) and \(4/5\). It is interesting to note that the marginal tax rates are increasing around up to the poverty line. As we noted earlier in more special case if the Pareto distribution holds throughout the range of \(n\) the marginal tax rates are increasing up to the poverty line. Now things are more complicated in this more general setting with income effects. As noted in the context of (21) the C-term tends to favour rising marginal rates when income is low. \(U_x\) has
also a central role in the term C. Income effects are related to the concavity of the utility of consumption as people are willing to work more when after tax income is lower. In the Charitable conservative case this means that the weights $\psi'U_x$ is decreasing both below and above poverty line and the weights jump to a lower value at $n_{pov}$. This suggests that this effect through $\psi'U_x$ with relative low $k$, the utility function (29) and lognormal $n$-distribution leads increasing marginal rates below the poverty line.

Figure 7 Charitable conservatism; $k=1/3$
4. Conclusion

The object of this paper has been, first, to clarify the interrelationships between Charitable Conservatism, Poverty Radicalism and Inequality Aversion. We have done this by showing how the patterns of welfare weights to which each of these gives rise are related to each other. Secondly, we have been concerned to demonstrate the implications of these different views for optimal income taxation. In terms of levels and patterns of marginal tax rates, modified Charitable Conservatism and Poverty Radicalism are on a continuum - at least in specifications we have used, and we would argue these to be the reasonable ones to choose - and by choice of low or high Inequality Aversion one can approximate either outcome fairly well. There would thus appear to be no fundamental qualitative difference between these three seemingly very different perspectives so far as their policy consequences are concerned. The difference, rather, is one of degree.
References
Rawls, J. (1982), Social unity and primary goods, in Utilitarianism and Beyond, eds. by A. Sen and B. Williams, Cambridge University Press, 159-85.